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Intervention Strategies to Improve Cognitive Processes of Children in Finding Solutions to Addition and Subtraction Problems

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INTERVENTION STRATEGIES TO IMPROVE COGNITIVE PROCESSES OF CHILDREN IN FINDING SOLUTIONS TO ADDITION AND SUBTRACTION PROBLEMS

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Introduction

The business of mathematics education in school is an extremely complex enterprise, influenced by interaction of countless learner, instructional and content variables. Studies that attempt to isolate individual factors for experimentation are routinely ignored by practitioners who judge them irrelevant amid the complexities of classrooms in action. Thus while organising concepts in the area of teaching and learning of specific type of mathematical contents, teachers commonly group topics into fields such as arithmetic, algebra, geometry, calculus etc. There has been research focussed on the special problems of teaching each of these curricular areas. However, topics in these separate fields often present similar structures as learning tasks such as - performance of a routine skills, state or apply conditions for use of a mathematical term, probe principles relating operations, construction of solutions to non routine problems etc.

Resnick and Ford (1980) have observed that "school mathematics in every curricular area involves learning of skills, concepts, principles and and problem solving. There have been major lines of research addressing the special conditions that influence each type of learning."

The area of computational skills i.e. abilities to add, subtact, multiply and devide numbers have long been central goals of school mathematics instruction. Research on teaching and learning of mathematical skill has been concentrated on the skills of arithmetic, in which the studies can be devided roughly into those that deal with basic facts and those that deal with the computational algorithms by which basic facts are applied to more complex calculations. Suydam and Dessart (1980) have analysed the research in these areas and found that 'despite many studies, more important questions than useful results.' Fey (1982) observed that on the learning of facts, the basic questions seem to be

a) what are the relative difficulties of various specific facts and operations?

b) what sequence and balance of role and meaningful instruction lead to best short-term or long-term learning?

c) What practice of drill pattern leads to greatest retention, and what strategies do students naturally use to retain facts?

d) How does the learning of facts for one operation enhance or interfere with that of the other operations?

Researches on these questions fall into roughly three periods of emphasis. For the first, the theory of arithmetic instruction was dominated by Thorndike's S-R approach to learning and research. Studies concentrated on determining which basic facts were most difficult and which patterns of drill and practice were most effective in fixing algorithmic behaviour. The next phase was reflected in the landmark work of Brownell (1947) which shifted the focus of arithmetic research by suggesting that meaning and understanding are essential factors even in the learning of skills. This view joined by Piagetian inspired interest in concrete bases for number learning, set the agenda for research on skills over past many years. The third major phase in arithmetic skill research lies largely in the future, as electroic calculators become available performance in arithmetic. This new technological environment raises fundamental questions about what skills are still important and how the technology can be used to assist in skill teaching (Fey, 1982).

However, in the area of researches in elementary school mathematics education in India, it seems revolving round the second phase which establishes that effective skill development can not be seperated from conceptual understanding and its use in problem solving. In the present study an attempt has been made to see what practices and drill patterns lead to greater retention and what common classroom intervention strategies help students to retain facts naturally with the following specific objectives in mind:

Objectives

i. to identify solution processes used by children,

ii. to diagnose learning difficulties of children,

iii. to develop remedial intervention strategies to overcome the learning difficulties of children, and

iv. to assess efficacy of teacher guided learning activities as intervention strategy,

in solving problems related to addition with carrying and subtraction with borrowing in elementary mathematics.

The present study has been delimited to the sub-competencies as given below in the area of addition and subtraction of class II mathematics delineated in an expert group meeting.

Sub Area: Addition

1. Addition with carrying
    - 1.1. Two digits with single digit
        - 1.1.1. sum at unit place is zero
        - 1.1.2. sum at unit place is not zero
    - 1.2. Two digits with two digits
        - 1.2.1. sum at unit place is zero
        - 1.2.2. sum at unit place is not zero
    - 1.3. two two-digits with one single-digit
        - 1.3.1. sum at unit place zero
        - 1.3.2. sum at unit place not zero
    - 1.4. Three two-digits numbers
        - 1.4.1. sum at unit place zero
        - 1.4.2. sum at unit place not zero

Sub Area: Subtraction

2. Subtraction with borrowing

2.1. Single digit from two digits

2.1.1. minimend's unit place not zero

2.1.2. minimend's unit place zero

2.2. Two digits with two-digits

2.2.1. minimends unit place not zero

2.2.2. minimends unit place zero

2.3. Difference in single digit

2.3.1. Unit place not zero

2.3.2. unit place zero

Oral and figural computation skills related to these competencies have been excluded in view of intricacies of assessement. Further, the study has been restricted to class II students studying in rural primary schools of a district under DPEP scheme.

Methodology

Design: The design of the study has been given in the following:

Stage	Presage	Process	Product
Stage 1	1.1. Competency based criterion referenced achievement test	1.1. Administration and itemwise scoring of the Achievement test	1.1. Identification of solution processes used 1.2. Identification of items/sub-competencies in which students were facing difficulties 1.3. Identification of students attempting wrong to items/sub-competencies labled difficult

Stage 2	2.1. Responses of students on Competency based criterion reference achievement test	2.1. Analysis of mistakes of students in different problem situations and identification of ways to attempt those problems correctly	2.1. Development of teacher guided learning activities as remedial interventions
Stage 3	3.1. Equivalent forms of Competency based criterion referenced achievement test on difficult sub-competencies	3.1. Administration of equivalent forms of achievement test on difficult sub-competencies on the identified group of students both pre and post remedial intervention sessions	3.1. Assessment of efficacy of remedial intervention strategy

Tools: A paper-pencil type competency based criterion reference achievement test consisting of 28 items (addition 16 and subtraction 12) was developed in an expert group meeting after carefully analysing the sub-competencies of addition with carrying and subtraction with borrowing alongwith problem situations of row, column and day to day life situation sums excluding oral and figural sums. In the test, students were directed to solve the question the way they wish in the given space provided with the question.

ii. Two equivalant forms of competency-based criterion-reference achievement tests were developed on those five sub competencies which were identified as most difficult in each of the addition with carrying and subtraction with borrowing. Each sub competency was represented in the achievement test.

iii. After careful analysis of the responses given by students in the competency based criterion referenced test the following teacher guided learning activities were identified.

For addition with carrying

a) Draw lines/figures, make bundles of five to add.

b) add first two addends, then add third one in the sum

c) note/write the carry at the top

d) write in terms of tens and ones then add

e) convert the row sums/figures in problem sums into column sums and follow any of the (a) to (d)

For subtraction with borrowing

a) use lines/figures to find difference

b) note the borrowing at the subtractor

c) write in terms of tens and ones then find difference

d) convert row/problem sums into column sums and follow any of the (a) to (c)

Sample and Collection of Data: The competency based criterion referenced test was administered to identify solution processes used, items/sub competencies most difficult and students with learning difficulties in these difficult items on a sample of 212 class II students from eight rural primary schools selected from one district (Dhenknal, Orissa) covered under DPEP scheme after ensuring that these concepts have already been tought in regular class by the teacher. This testing situation was designated as diagnostic testing situation. In this administration schools were selected randomly while all the students present in the class on the date of data collection were administered the test

Itemwise scoring of response sheets of each student enabled to identify five items each in addition with carrying and subtraction with borrowing which were attempted wrong by majority of students. The item No., sub-competency involved, the problem situation and the percentage of students attempting it wrong in the order of their rank both in addition with carrying and subtraction with borrowing have been given below in Table A and B.

Table A

Difficult Sub-Competencies in Order of Their Rank on Addition with Carrying

Sl. No.	Item No.	Sub-Competency	Problem Situation	% of students attempted wrong
1	8	Addition of three two-digit numbers when the sum at unit place is not zero	Column sum	55.66
2	6	Addition of two two-digit numbers with one single-digit number when the sum at unit place is not zero	Column sum	45.75
3	16	Addition of three two-digit numbers when the sum at unit place is zero	Problem sum	44.81
4	11	Addition of two two-digit numbers with one single-digit number when the sum at unit place zero	Row sum	42.92
5	12	Addition of three two-digit numbers when the sum at unit place is not zero	Row sum	41.98

Table B

Difficult Sub-Competencies in Order of Their Rank on Subtraction with Borrowing

Sl.No.	Item No.	Sub-Competency	Problem Situation	% of students attempted wrong
1	4	Subtraction of two digit number from two digit number when unit place of minimend has zero	Column sum	40.09
2	9	Subtraction of two digit number from two digit number when unit place of minimend has zero	Row sum	37.26
3	6	Subtraction when difference is in single digit and unit place has zero	Column sum	35.38
4	12	Subtraction when difference is in single digit and unit place has zero	Problem sum	32.55
5	5	Subtraction of two digit number from two digit number unit place of minimend has not zero	Column sum	31.60

After that, students who have attempted more than 60 percent items wrong of these five most difficult items in addition with carrying and subtraction with borrowing were identified. Keeping in view the intricacies of administration of remedial testing materials, only 50 students from three schools were retained for remedial intervention.

In each of the three schools, students identified with learning difficulties were categorised into two groups. Group I included those students who have attempted more than 60 percent of difficult items wrong in addition with carrying. This group was administered equivalent form (form A) of achievement test on addition with carrying. Group II included those students who have attempted more then 60 percent of difficult items womg in subtraction with borrowing. This group was administered equivalent form (form A) of achievement test on subtraction with borrowing. After that, both the groups were given exposure and practice under the supervision of the researcher in teacher guided learning activities to solve difficult items from the respective area. Researcher also helped students to use these activities in simple ways to solve the specific problems given. Next day the equivalent form (form B) of the achievement tests were administered in respective groups. Both the equivalent forms of achievement tests were scored. A sample of 47 (26 for addition with carrying and 21 for subtraction with borrowing) was procured.

Statistical Techniques: Simple percentage analysis, t-test for significance of difference between correlated means following difference method and significance of difference between correlated percentages were applied for analysis of data and interpretation of results.

Findings

1. Majority of students have used cognitive process of mental computation followed by process of converting the row sums and problem sums into vertical forms and few students have used figures/lines to find out answers to the problems both on addition with carrying and subtraction with borrowing as can be seen in the following Table 1.

Table-I

Cognitive Processes Used by Students in Solving Difficult Items on Addition with Carrying and Subtraction with Borrowing

Sub Area	Item No.	Cognitive Processes Used		
		Mental	Figural	Vertical Representation
Addition with Carrying	6	94	5	-
	8	110	5	-
	11	80	-	9
	12	63	-	25
	16	52	-	50
Subtraction with Borrowing	4	78	5	-
	5	62	5	-
	6	77	3	-
	9	72	1	9
	12	27	1	67

2. There was statistically significant gain in achievement scores of students on five difficult items on addition with carrying. The gains, between pre-intervention and post-intervention testing as well as diagnostic testing and post-intervention testing were significant at 01 level and between diagnostic testing and pre-intervention testing was significant at .05 level. (refernce Table 2).

Table 2

Significance of Gain in Achievement of Students on Difficult Items on Addition with Carrying During Diagnostic Testing, Pre-Intervention Testing and Post-Intervention Testing

Values	Difference in Scores on Five Difficult Items on Addition with Carrying		
	D_{12}	D_{23}	D_{13}
Sample size	26	26	26
M_D	0.88	1.77	2.65
SD_D	2.01	2.67	2.97
t-value	t_{12}=2.26*	t_{23}= 3.40**	t_{13}=4.57**

* Significant at .05 level

** Significant at .01 level

This can clearly be seen from the Table 2 that maturation has contributed significantly though at .05 level of significance, remedical intervention has contributed significantly at .01

level of significance and maturation coupled with remedial intervention has contributed significantly at .01 level of significance to the mean achievement of students in difficult sub-competencies on addition with carrying. The term maturation refers to the enrichment acquired in the competency by the student in school or otherwise during the intervening period.

3. There was statistically significant gain at .01 level in achievement scores of students on five difficult items on subtraction with borrowing between all the three testing situation i.e. diagnostic testing, pre-intervention testing and post-intervention testing (reference Table 3).

Table 3
Significance of Gain in Achievement of Students on Difficult Items on Subtraction with Borrowing During Diagnostic Testing, Pre-Intervention Testing and Post-Intervention Testing

Values	Difference in Scores on Five Difficult Items on Addition with Carrying		
	D_{12}	D_{23}	D_{13}
Sample size	22	22	22
M_D	1.67	2.14	3.81
SD_D	2.52	2.97	4.11
t-value	t_{12}=3.03**	t_{23}= 3.29**	t_{13}=4.25**

** Significant at .01 level

It can clearly be seen from the Table 3 that maturation, remedial intervention and maturation coupled with remedial intervention has significantly contributed in the mean achievement scores of students with in difficult sub-competencies on subtraction with borrowing. The term maturation refers to the enrichment acquired in the competency by the student in school or otherwise during the intervening period.

4. The difference between percentage of students attempting difficult items on addition with carrying correct during diagnostic, pre-intervention and post-intervention testing along with the corresponding t-values for significance of difference have been given in Table 4 are self explanatory.

Table 4

Significance of Difference Between Correlated Percentages of Students Attempting Difficult Items of Addition with Carrying Correct During Diagnostic Testing, Pre-Intervention Testing and Post-Intervention Testing (N-26)

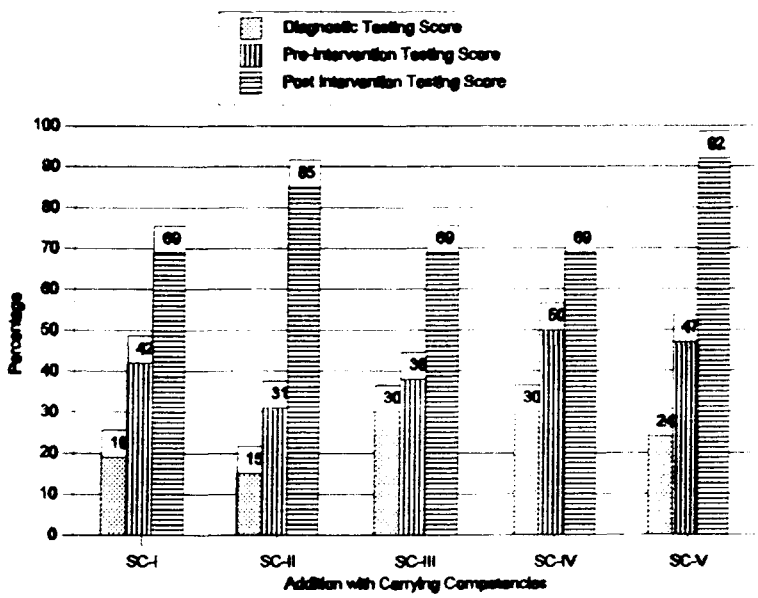
Sl.No.	Sub-Competency/Item No.	Percentage of Students Attempting Correct			Significance of Difference Between Correlated Precentages		
		Diagnostic Testing (1)	Pre-Intervention Testing (2)	Post-Intervention Testing (3)	t_{12}	t_{23}	t_{13}
1	I/6	19.23	42.31	69.23	2.11*	1.87	3.36**
2	II/8	15.38	30.77	84.61	1.20	3.59**	4.37**
3	III/11	30.77	38.46	69.23	0.66	2.26*	2.47*
4	IV/12	38.46	50.00	69.23	0.94	1.28	2.08*
5	V/16	23.08	46.16	92.31	1.72	3.54**	4.27**

* Significant at .05 level

** Significant at .01 level

The corresponding bar diagrams have also been given for better comprehension in Figure I.

Fig. 1: Bar Diagrams Representing Percentage of Students Attempting Correct the Items Identified on Addition with Carrying during Diagnostic Testing, Pre-Intervention & Post-Intervention Testing



It is clear from the values given in the Table 4 and Figure I that maturation alone has not improved significantly the proportion except in case of item no. 6, where it has improved significantly at .05 level. Whereas remedial intervention has improved significantly the proportions in item no. 11 at .05 level and items No. 8 and 16 at 01 level. Maturation and intervention together has significantly improved the proposition in all the five items on addition with carrying viz. in case of items 11 and 12 at .05 level and items 6,8 and 16 at .01 level.

5. The difference between percentages of students attempting difficult items on subtraction with borrowing correct during diagnostic, pre-intervention and post-intervention testing along with the corresponding t-values for significance of difference have been given in Table 5, are self-explanatory.

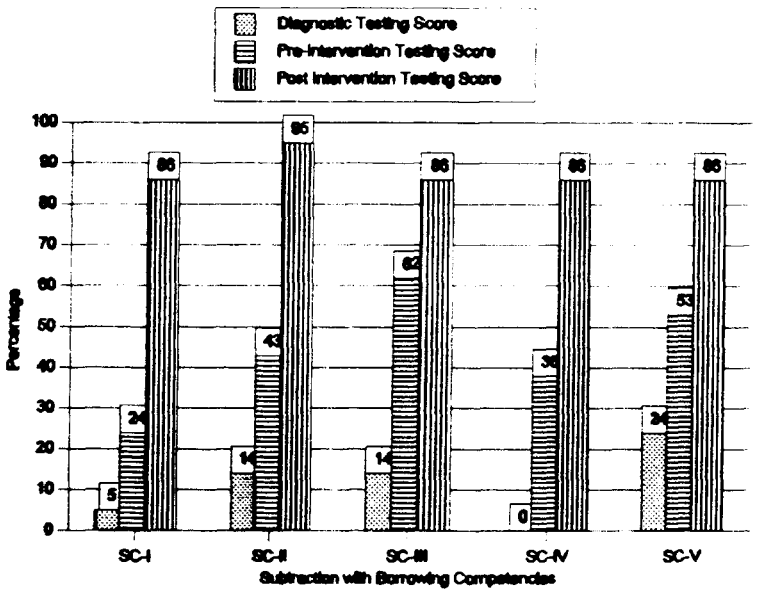
Table 5
Significance of Difference Between Percentages of Students Attempting Difficult Items of Subtraction with Borrowing Correct During Diagnostic Testing, Pre-Intervention Testing and Post-Intervention Testing (N=21)

Sl.No.	Sub-Competency/Item No.	Percentage of Students Attempting Correct			Significance of Difference Between Precentages		
		Diagnostic Testing (1)	Pre-Intervention Testing (2)	Post-Intervention Testing (3)	t_{12}	t_{23}	t_{13}
1	I/4	4.76	23.81	85.71	2.00	3.61**	4.13**
2	II/5	14.29	42.86	95.24	1.94	3.30**	4.13**
3	III/6	14.29	61.90	85.71	3.18**	1.52	3.89**
4	IV/9	0.00	38.10	85.71	2.84*	2.84*	4.26**
5	V/12	23.81	52.38	85.71	1.94	2.31*	3.35**

* Significant at .05 level
** Significant at .01 level

The corresponding bar diagrams have also been given for better comprehension in Figure II.

Fig. II : Bar Diagram Representing Percentage of Students Attempting Correct to the Items Identified on Substraction with Borrowing during Diagnostic Testing, Pre-Intervention and Post-Intervention Testing



It is clear from the values given in the Table 5 and Figure II above that maturation alone has improved significantly the proportions in case of items no. 9 and 6 at .05 and .01 levels respectively whereas remedial intervention has significantly improved the proportions in case of item no 9 and 12 at .05 level and item no. 4 and 5 at .01 level. In case of item no. 6 it has not improved significantly. Maturation and remedial intervention together has significantly improved the proportion in all the five items on subtraction with borrowing at .01 level of significance.

6. Scruitiny of the equivalent forms of the achievement test administered both before and after the intervention revealed that students have utilised the techniques learned during remedial intervention in the form of teacher guided learning activities in finding the solutions to most of the problems on addition with carrying and substraction and with borrowing. However, few students have used these in solving few problems during pre-intervention testing also.

7. Teacher guided learning activities as remedial intervention strategy has helped in achieving mastery (80 percent or more marks) to about 69 percent students in addition with carrying and to 86 percent students in subtraction with borrowing. However maturation has helped 27 percent and 29 percent students in attaining mastery in addition with carrying and subtraction with borrowing respectively. These figures have been given distinctly in the following Table 6.

Table 6
Number and Percentage of Students Attainig Mastery in Competencies on Addition with Carrying and Subtraction with Borrowing During Pre Intervention and Post Intervention.

Competency	Diagnostic Testing	Pre-Intervention Testing	Post Intervention Testing
Addition with Carrying (N=26)	-	07 (27%)	18 (69%)
Subtraction with borrowing (N=21)	-	06 (29%)	18 (86%)

Implication of Findings

Child reconstructs acquired knowledge continuously in familiar situations in various ways. But this alone can not lead to mastery in case of every competency. That is why enrichment of acquired knowledge of the concept in various ways in classes and in other activities has no doubt brought improvement but significant improvement in very few 30% sub-competencies. To further improve upon acquisition of the competency to the level of mastery, suitable interventions are required. If these interventions are designed in such a way that learner's pre-requisites are properly taken care of, these can further improve the acquisition of competencies. Teacher guided learning activities derived and designed from learner's solution processes as intervention strategies have brought significant improvement in acquisition of very large number 70% of sub-competencies. Since in a process of education, purposeful remedial interventions can not be seperated from enrichment of acquired knowledge continuously in familier situations in various ways (maturity), both these put

together have significantly improved the solution processes of children to achieve mastery in almost all difficult sub-competencies.

On the basis of the findings of the study it can be concluded that appropriate remedial intervention strategies such as teacher guided learning activities in lower classes help in bringing improvement in the cognitive processes of children to the extent of attaining mastery over the competencies in a specific subject area. While deciding about appropriateness of various intervention strategies it must be remembered that children actively construct knowledge for themselves through interaction with the environment and reorganisation of their own mental constructs. Referring the research on addition and subtraction Romberg and Carpenter (1986) too suggest that 'the current primary mathematics curriculum fails to capitalise on rich informal mathematics that children bring to instruction. Children's invented strategies for solving addition and subtraction problems are frequently more efficient and more conceptually based than the mechanical procedures, included in many mathematics programmes'.

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