A STUDY OF PERFORMANCE ANALYSIS OF STUDENTS IN MATHEMATICS

(Class XII)

March, 1998 Examination

CENTRAL BOARD OF SECONDARY EDUCATION

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Preface

It is now increasingly realised that in addition to serving the purpose of certification and classification, examinations can also be used as powerful tools for improving teaching/learning process. The analysis of students' responses to different questions asked in the examination can serve to diagnose the weaknesses in learning as well as shortcomings in curriculum transaction. Keeping in view the potential and importance of such an analysis, Central Board of Secondary Education undertook the present study in the subject of Mathematics for class XII for March, 1998 examination.

The study attempts to focus on highlighting specific errors and mistakes committed by majority of students while writing responses to different questions asked in the question papers. The performance of students in respect to each question in the question paper used for Delhi Region as well as outside Delhi Region has been analysed. It includes qualitative as well as quantitative aspects of responses. It is hoped that questionwise and topicwise suggestions for remedial action will be put to best use for maximising students' performance in the subject. We hope the study will provide useful feedback to the schools.

The Board is thankful to the members of working group and Resource Persons for their valuable contribution to the study. I compliment Shri G. Balasubramanian, Director (Academic), CBSE who helped to conceive and design the study. Shri R.P. Sharma, Education Officer (Science) deserves appreciation for coordinating the project, carrying out its related activities and bringing out this publication.

New Delhi
February, 1999

B.P. Khandelwal
Chairman

Introduction

The need and importance of objective based curriculum, its effective transaction and the evaluation of achievement of learners vis-a-vis the expected learning outcomes cannot be denied. The three components form the backbone of an effective educational system. Out of these three components, the evaluation of learners' achievement measures the extent to which the first two components have been taken care of and thus acts as a self-regulating quality control device.

Keeping in view the potential and significance of analysing students' responses to different questions asked in the examinations, the Central Board of Secondary Education undertook the present study in the subject of Mathematics for class XII candidates who appeared in March, 1998 examination.

OBJECTIVES OF THE STUDY

The performance analysis of the selected sample of candidates appearing in March 1998 examination has been undertaken with a view to :

(i) identify the type and nature of errors committed by candidates as against ideal responses given in the marking scheme.

(ii) diagnose specific areas of learning difficulty in different topics.

(iii) identify possible shortcomings in curriculum transaction.

(iv) provide feedback to schools for initiating necessary actions to imporve students' achievement.

(v) suggest remedial measures to be taken by subject teachers and schools to improve students' understanding of the subject.

METHODOLOGY FOLLOWED

The quality of answers to a question by different students may be affected by number of variables such as type and location of a school, the quality of learning experiences, the availability of resources and the type of question etc. Whatever may be the background of a student appearing in the examination, his/her overall achievement in a subject is judged by the aggregiate marks he/she obtains. To cover candidates getting all ranges of marks is one of the important aspects of getting a true picture of prevailing conditions.

(i) SELECTION OF ANSWER BOOKS

The study was carried out for Delhi Region as well as outside Delhi Regions. It was decided to analyse 100 answer scripts from each category.

The method adopted for selection of answer books was stratified random sampling so that the answer books were mixed in nature and were independent of any particular parameter. Since the study was to be conducted in a given time frame, only the first question paper in both the categories was considered for the purpose of analysis. Mark-wise distribution of selected sample of answer scripts was as under

Strata	Range of Marks	Number of Delhi Reg.	answer-scripts Outside Reg.
I.	0-33	15	15
II.	34-50	35	35
III.	51-75	35	35
IV.	76-100	15	15
	Total	100	100

It may be noted that bulk of answer-scripts were taken for the marks ranging 33% to 75% so as to focus the study on average students.

(ii) QUESTION PAPERS USED

Although there were three sets each of separate question papers for Delhi Region and outside Delhi Region it was decided to consider only those answer books in which the candidates had used the first paper of the sets for both the regions. Consequently, for Delhi Region, paper 65/1/1 was taken and for outside Delhi region, question paper of code 65/1 was used.

(iii) SELECTION OF WORKING GROUP MEMBERS

The working group for the study consisted of those competent and dependable subject experts, teachers educators, who usually act as examiners, additional head examiners and head examiners, paper setters and are well conversant with the teaching/ learning of the subject at the school stage. Some subject specialists from NCERT and Principals/Vice-Principals, experienced teachers were also included in the working group as resource persons.

The working group was divided into six sub-groups, each sub-group handling a separate unit of the syllabus. Each sub-group had two persons who were assigned the question nos. relating to the unit in two papers 65/1/1 and 65/1. They went through the answere books of examinees and recorded their observations in the prescribed proforma. Each sub-group was responsbile for preparing a question wise report of the questions alloted to them for two papers.

(iv) PROFORMAS USED

Different proformas required for the study were developed in a separate workshop prior to the actual exercise on performance analysis. The first proforma was used for analysis of types of specific errors whereas the second was related to statistical data. Detailed questionwise report was prepared on the third proforma.

The results of the study alongwith the statistical analysis are presented in the next section of the report.

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DELHI REGION

Q. No. 1 By using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Row Transformation on Matrices

Expected Answer

$$\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$$

$$R_2 \to R_2 - 2R_1$$

$$\begin{pmatrix} 1 & 2 \\ 0 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$R_2 \to -\frac{1}{5}R_2$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{pmatrix} A$$

$$R_1 \to R_1 - 2R_2$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{pmatrix} A$$

$$\therefore A^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{pmatrix} \text{ or } \frac{1}{5}\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
11	28	21	9	5	26	0.90

General Remarks

89% candidates from the sample attempted this question, out of which about 24% committed conceptual errors. Only 26% candidates from the sample could do the question correctly, getting 2 marks whereas 32% of them got zero mark. About 60% of the candidates followed desired method but could not complete the same because of computational errors.

Errors committed with examples

(i) Symbol of determinant has been used in place of symbol of a matrix by some of the students.

(ii) Inverse has been found by adjoint method instead of using row transformation.

(iii) Indication of transformations has been done wrongly. For example, though $R_1 \to R_1 - R_2$ has been used but the indication has been given for $R_2 \to R_2 - R_1$

(iv) Some candidates have not written A in the step A = IA

(v) Calculation errors of the type :

$$2 - (-2) = 0 \text{ and } 2 + (-3) = 5$$

Probable causes

(i) The candidates are confused about the notations for determinants and matrices.

(ii) The candidates appear to be totally mixed up with row/column transformations. They mean something else but carry out something different.

Suggested Remedial Measures

(i) The students should be made to understand the diffrence between row and column transformations. They should be told specifically that only row transformations are to be used as asked in the paper.

(ii) The operations should be indicated clearly and followed faithfully.

(iii) Use of notations should be clearly explained to the students. They should be specifically told that ( ) and [ ] notations are used for matrices and | | is used for a determinant.

Q. No. 2 : Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Type of Question	Marks	Objective	Concept/Sub concept
VSA	2	Understanding	To express the matrix as the sum of symmetric and skew-symmetric matrix.

Expected Answer :

$$A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \quad \text{Also } A = \underbrace{\frac{1}{2}(A + A')}_{\text{Symmetric}} + \underbrace{\frac{1}{2}(A - A')}_{\text{Skew Symmetric}}$$

Symmetric $\frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & -3 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$

Skew - Symmetric $\frac{1}{2}(A - A') = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 3 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
22	12	9	12	10	35	1.31

General Remarks

78% of the candidates from the sample attempted the question but only 44% could do it correctly to the end. 22% candidates did not attempt the question at all. It indicates that the co. cept of symmetric and skew-symmetric matrix is not clear to a majority of candidates.

Errors committed with example

(i) The candidates do not understand the concept of transpose of a matrix. The question has been done haphazardly. Some of them have interchanged rows whereas others tried changing columns only.

(ii) Some of the candidates have wrongly written the relationship as

$$A = \frac{1}{2}(A + A') - (A - A')$$

(iii) Some candidates do not seem to have the idea of symmetric and skew-symmetric matrices because the matrices they have expressed are neither symmetric nor skew-symmetric.

(iv) Computational errors in adding the corresponding elements of matrices.

Probable causes

(i) The concept of transpose of a matrix is not clear.

(ii) The concept of symmetric and skew-symmetric matrix is not clear.

Suggested Remedial Measures

(i) Sufficient practice is required for finding the transpose of a given matrix.

(ii) The concept of symmetric and skew-symmetric matrices should be made clear to the students.

(iii) Sufficient practice needs to be given to students to express a matrix as a sum of a symmetric and a skew-symmetric matrix. This will help the students in understanding the concepts as well as help them in minimising computational errors.

Q. No. 3. Show that the vectors $\hat{i}-2\hat{j}+3\hat{k}, -2\hat{i}+3\hat{j}-4\hat{k}$ and $\hat{i}-3\hat{j}+5\hat{k}$ are coplanar.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	02	Knowledge	Scalar Triple Product/ Coplanarity

Expected Answer :

Condition of coplanarity of three vectors $\bar{a}, \bar{b}, \bar{c}$ is $[\bar{a}, \bar{b}, \bar{c}] = 0$

$$[\bar{a}, \bar{b}, \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(15-12) + 2(-10+4) + 3(6-3)$$
$$= 3 - 12 + 9$$
$$= 0$$

Hence the vectors are coplanar.

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
11	17	2	6	1	63	1.51

General Remarks

89% of the candidates from the sample attempted the question and about 70% of them could do it correctly. 20% of those who attempted, got zero mark as they confused the concept of coplanarity of vectors with collinearity of points.

Errors committed with examples

(i) Some of the candidates took vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ as position vectors of four points and then tried to show that points are collinear.

(ii) Others took $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ as points, found AB, BC, CD and then used

$$\frac{1}{2}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

Probable causes

(i) Concept of coplanarity of vectors is not clear to the candidates.

(ii) Difference between coplanarity and collinearity is not clear to many of them.

(iii) Scaler triple product of three vectors gives the volume of the parallelopiped formed by three vectors and if volume is zero, it implies that vectors are coplanar. This concept is not clear to many students.

Suggested Remedial Measures

(i) Difference between coplanarity of vectors and Collinearity of points must be made clear to the students by taking sufficient number of examples.

(ii) Correct formula for volume of parallelopiped and what happens when volume becomes zero must be clarified to the students by taking examples.

Q. No. 4. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, prove that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$ provided $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Knowledge	Cross product and parallel vectors

Expected Answer :

$\because \vec{a}\times\vec{b}=\vec{c}\times\vec{d} \Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{d}=\vec{0}$ ....(i)

& $\vec{a}\times\vec{c}=\vec{b}\times\vec{d} \Rightarrow \vec{a}\times\vec{c}-\vec{b}\times\vec{d}=\vec{0}$ ....(ii)

From (i) & (ii) $\vec{a}\times\vec{b}-\vec{c}\times\vec{d}=\vec{a}\times\vec{c}-\vec{b}\times\vec{d}$

or $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}$

$\vec{a}\times(\vec{b}-\vec{c})=(\vec{c}-\vec{b})\times\vec{d}$

or $\vec{a}\times(\vec{b}-\vec{c})-(\vec{c}-\vec{b})\times\vec{d}=\vec{0}$

$\vec{a}\times(\vec{b}-\vec{c})+(\vec{b}-\vec{c})\times\vec{d}=\vec{0}$

$\Rightarrow$ $\vec{a}\times(\vec{b}-\vec{c})-\vec{d}\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow$ $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=\vec{0}$

$\Rightarrow$ $(\vec{a}-\vec{d})\,//\,(\vec{b}-\vec{c})$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	o
44	31	4	5	1	15	0.70

General Remarks

75% candidates from the sample either did not attempt this question or obtained zero mark. Only 25% of those who attempted could do it correctly scoring full marks. Others committed conceptual errors.

Errors committed with examples

(i) Most of the candidates could not proceed further after writing $\bar{a}\times\bar{b}-\bar{c}\times\bar{d}=o$ and $\bar{a}\times\bar{c}-\bar{b}\times\bar{d}=o$

(ii) Some candidates had written $(\bar{a}-\bar{d})\times(\bar{b}-\bar{c})$ but could not use the given results to get it equal to zero

Probable causes

(i) The concept of $\bar{a}\times\bar{b}=-\bar{b}\times\bar{a}$ is not clear to significant number of the candidates.

(ii) The concept of $\bar{a}\times(\bar{b}+\bar{c})=\bar{a}\times\bar{b}+\bar{a}\times\bar{c}$ is not clear to many of them.

Suggested Remedial Measures

Sufficient practice should be given to the students to make it clear that the cross-product of vectors is not commutative but is distributive over addition, while dot product is commutative as well as distributive.

Adequate pracitice and examples should be given to the students to prove that $\bar{a}\times\bar{b}=-\bar{b}\times\bar{a}$

Q. No. 5. Verify Rolle's Theorem for the function $f(x)=x^2-x-6$ in the interval $[-2,3]$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Rolls Theorem

Expected Answer :

1. $f(x)$ is continuous, being a polynomial function
2. $f'(x) = 2x - 1$ exists in $]2, -3[$, $\therefore f(x)$ is derivable
3. $f(-2) = 4 + 2 - 6 = 0$, $f(3) = 9 - 3 - 6 = 0 \Rightarrow f(-2) = f(3) = 0$

$\therefore$ Some $c \in]-2, 3[$ Such that $f(c) = 0$

$\Rightarrow f'(c) = 2c - 1 = 0 \Rightarrow C = \frac{1}{2}$

$c = \frac{1}{2} \in]-2, 3[\Rightarrow$ Rolle's Theorem is verified.

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
2	5	3	22	14	54	1.56

General Remarks :

This question has been attempted by majority of candidates (98%) though only 55% could complete it correctly getting full marks. 44% of the candidates who attempted the question could not do it because of conceptual and computational errors.

Errors committed with examples

(i) All conditions of Rolle's Theorem are neither stated nor verified.

(ii) $c \in$ Domain is not clearly mentioned.

(iii) $f'(c) = 0 \Rightarrow f(c) = 0$, $\therefore c^2 - c - 6 = 0 \Rightarrow c = -2, 3$

(iv) $f(x) = x^2 - x - 6$

$f'(x) = 2x - x$

Probable causes

(i) Majority of the candidates are not clear about 3 conditions of Rolle's Theorem.

(ii) Conditions are mentioned without any verification which reflects rote learning in place of understanding.

(iii) The concept of open interval and closed interval is not clear to many of them.

Suggested Remedial Measures

(i) Emphasis should be given to verification of conditions of Rolle's theorem.

(ii) The domain of C should be mentioned.

(iii) Computational errors should be avoided while finding the value of C.

(iv) The concept of open interval and closed interval should be made clear to the students with the help of examples.

Q. No. 6. Evaluate $\lim\limits_{x \to \pi} \frac{\sin x}{x - \pi}$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Limit

Expected Answer

$$\lim_{x \to \pi} \frac{\sin x}{x - \pi}$$

Putting $x - \pi = y$ so that when $x \to \pi,\ y \to 0$

$$\therefore \lim_{x \to \pi} \frac{\sin x}{x - \pi} = \lim_{y \to 0} \frac{\sin(\pi - y)}{-y}$$

$$= \lim_{y \to 0} \frac{\sin y}{-y}$$

$$= -1$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
15	42	4	9	1	29	0.84

General Remarks

85% of the candidates from the selected sample attempted this question out of which about 50% secured zero mark. Of those who attempted the question, 34% completed it correctly thus getting 2 marks. Others either committed computationall or conceptual errors.

Errors committed with examples

(i) $\lim_{x \to \pi} \frac{\text{Sin } x}{x - \pi} = \lim_{x \to \pi} \text{Sin } x . \lim_{x \to \pi} \frac{1}{(x - \pi)} = \lim_{x \to \pi} \frac{\text{Sin } x}{x} . x . \lim_{x \to \pi} \frac{1}{x - \pi}$

(ii) $\lim_{x \to \pi} \frac{\text{Sin } x}{x} . \frac{x}{x - \pi} = \lim_{x \to \pi} \frac{\text{Sin } x}{x} . \lim_{x \to \pi} \frac{x}{x - \pi}$

$= 1.0 = 0$

Probable causes

(i) The concept of limits is not clear to the candidates.

(ii) Many of the candidates are confused with the limit

$\lim_{x \to 0} \frac{\sin x}{x}$ with $\lim_{x \to \pi} \frac{\sin x}{x}$

Suggested Remedial Measures

The concept of limit should be made more clear to the students with the help of different examples. Sufficient practice should be given to them to change various given limits to standard form with the help of substitution.

Q. No. 7. Differentiate $\tan^{-1}\left[\frac{\cos x}{1 + \sin x}\right]$ w.r.t. x.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Differentiation of inverse trigonometric functions

Expected Answer : $y = \tan^{-1}\left[\frac{\cos x}{1+\sin x}\right]$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{(\cos\frac{x}{2} + \sin\frac{x}{2})^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan^{-1}\frac{x}{2}}{1 + \tan^{-1}\frac{x}{2}}\right]$$

$$= \tan^{-1} \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
17	34	6	10	2	31	0.94

General Remarks

The question has been attempted by as many as 83% candidates though only 31% could do it correctly. The average score of this question is far below expectations.

Errors committed with examples

(i) $y / = \frac{1}{1+\left(\frac{\cos x}{1+\sin x}\right)^2}$

(ii) $\cos x = 1 - 2\sin^2 x$ or $1 + 2\sin^2 \frac{x}{2}$

(iii) $\frac{d}{dx} \tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] = \tan^{-1}\left[\frac{-\sin x}{\cos x}\right] = \tan^{-1} \tan x\,(-1) = -1$

(iv) $y = \tan^{-1}(\sec x - \tan x) = \tan^{-1}\left[\sec x - (1 - \sec^2 x)\right]$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right] = \frac{x}{2} - \frac{x}{2}$$

$$y' = -\frac{1}{2}$$

(v) $\tan^{-1} x = \frac{1}{\tan x}$

Probable causes

(i) Lack of knowledge of trigonometric formulae.

(ii) The concept of inverse function is not clear.

(iii) Differentiation of composite function is not clear to many.

Suggested Remedial Measures

(i) Sufficient practice should be given so that students can recall and apply standard trigonometric formulae appropriately.

(ii) Sufficient practice should be given to differentiate composite functions.

(iii) The difference between $\tan^{-1}\theta$ and $\frac{1}{\tan\theta}$ should be made clear. The concept of trigonometric inverse function should be made very clear so that the students do not committ above mentioned errors.

Q. No. 8. Evaluate : $\int \frac{(\log x)^2}{x} dx$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	02	Knowledge	Integration by substitution

Expected Answer : Let log x = t

$$\frac{1}{x}\,dx = dt$$

$$\therefore\ I = \int \frac{(\log x)^2}{x}\,dx = \int t^2\,dt = \frac{t^3}{3}$$

$$\therefore\ I = \frac{1}{3}(\log x)^3 + c$$

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
09	14	–	01	02	74	1.82

General Remarks

91% candidates from the sample attempted this question. Out of those who attempted, about 81% did it correctly getting full marks whereas 6% did not give complete steps of solution and others committed either computational or conceptual errors.

Errors committed with examples

(i) The candidates committed mistakes in selecting proper substitution. For example, some students took $x = \frac{1}{t}$ in place of log x = t

(ii) The answer was left in terms of t only and was not converted back in x.

(iii) The derivative of $\frac{1}{x}$ was taken as log x

Probable causes

(i) Many candidates are not clear about laws of logarithm.

(ii) Many of them lack the practice of choosing proper substitution which leads to wrong answer.

(iii) Lack of basic idea of derivative of functions

Suggested Remedial Measures

(i) Sufficient practice should be given to the students about the use of laws of logarithms in problems.

(ii) Candidates should be given sufficient practice in finding derivatives of functions in which variable occurs in the denominator.

(iii) Directions related to choosing proper substitution at proper places and giving the answer free from substituted variable should be emphasised upon.

Q. No. 9 : Evaluate : $\int \frac{x\ e^x}{(x+1)^2}\ dx$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	02	knowledge	Integration using integration by parts or the formulae for $\int (f(x) + f'(x))e^x\ dx$

Expected Answer :

$$\int \frac{x\ e^x}{(x+1)^2}\ dx = \int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] e^x\ dx$$

$$= \frac{e^x}{x+1} + c \qquad \left[\text{using} \int [f(x) + f'(x)]e^x\ dx = f(x)e^x + c\right]$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
21	34	05	04	02	34	0.98

General Remarks :

79% of the candidates from the sample attempted this question. Out of those who attempted, 43% did it correctly getting full marks. 13% candidates did not give complete steps of solution and others either committed computational or conceptual errors.

Errors committed with examples

(i) Some candidates could not identify the form to which the question was to be reduced before applying the result of the standard form.

(ii) Some other candidates who used integration by parts, committed mistake in the application of method.

(iii) Some other candidates got mixed up with f(x) and f'(x)

Probable causes

(i) Candidates lack practice in identifying integrals which can be reduced to standard form after some adjustments or calculations.

(ii) The method of "integration by parts" is not clear to many candidates.

Suggested Remedial Measures

(i) Sufficient examples should be given to the students to put those expressions in standard form where direct formulae can be used.

(ii) Practice should be given to the candidates to do problems "by parts" and verify their answers.

(iii) It should be explained to the students that a lot of labour can be saved if the function, with a little effort, can be brought to the standard form where the formulae can be directly used.

Q. No. 10. Evaluate

$$\int_0^8 |x-5|\, dx$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Use of property $\int_0^8 f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Expected Answer

$$\int_0^8 |x-5|\,dx = \int_0^5 (5-x)\,dx + \int_5^8 (x-5)\,dx$$

$$= \left[\left(5x - \frac{x^2}{2}\right)\right]_0^5 + \left[\left(\frac{x^2}{2} - 5x\right)\right]_5^8$$

$$= \frac{25}{2} - 8 + \frac{25}{2}$$

$$= 17$$

Performance level of students

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
12	27	06	17	05	33	1.06

General Remarks :

88% of the candidates from the sample attempted this question. Out of those who attempted, 38% did it correctly. 43% candidates committed conceptual errors while 17% committed computational errors.

Errors committed with examples :

$$\int_0^8 |x-5|\,dx = \int_0^8 |8-5|\,dx$$

$$\int_0^8 |x-5|\,dx = \int_0^8 (x-5)\,dx + \int_5^8 (x-5)\,dx$$

$$\int_0^8 |x-5|\,dx = \int_0^4 (x-5)\,dx + \int_0^4 (5-x)\,dx$$

$$\int_0^8 |x-5|\,dx = \int_0^8 (x-5)\,dx$$

Probable causes :

(i) The concept of modulus

$|x| = x$ for $x \geq 0$

$= -x$ for $x < 0$ is not clear to the candidates.

(ii) Application of the property $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ is not clear to many.

Suggested Remedial Measures :

(i) The concept of modulus $|x| = x$ for $x \geq 0$ and $= -x$ for $x < 0$ should be made very clear by taking suitable examples.

(ii) The difference between question with modulus sign and without modulus sign should be made very clear to the students.

(iii) Sufficient practice be given to candidates to use the property.

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Q. No. 11. : Evaluate :

$$\int \frac{dx}{\sqrt{2-4x+x^2}}$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	02	Understanding	Integration (use of the formulae for $\int \frac{dx}{\sqrt{x^2-a^2}}$)

Expected Answer :

$$\int \frac{dx}{\sqrt{2-4x+x^2}} = \int \frac{dx}{\sqrt{(x-2)^2-(\sqrt{2})^2}}$$

$$= \log\left|(x-2)+\sqrt{x^2-4x+2}\right|+c \qquad \left[\text{Using formula for } \int \frac{dx}{\sqrt{x^2-a^2}}\right]$$

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
17	11	02	14	05	51	1.50

General Remarks :

83% candidates in the sample attempted this question. Out of those who attempted 61% did it correctly getting full marks. 12% candidates did not give complete steps of solution. Others either committed calculation errors or conceptual errors.

Errors committed with examples :

(i) Some candidates have either not been able to convert the denominator of the integral as sum or difference of two squares or have done it wrongly.

(ii) The formula $\int \frac{dx}{\sqrt{x^2 - a^2}}$ was not properly used. Instead, the standard result of $\int \frac{dx}{x^2 - a^2}$ was written by some candidates as answer.

Probable causes :

(i) Many candidates lack practice in writting quadratic expressions as sum or difference of two squares.

(ii) The candidates should carefully read the question and identify the integral as $\int \frac{dx}{\sqrt{x^2 - a^2}}$ or $\int \frac{dx}{x^2 - a^2}$ and then proceed further.

Suggested Remedial Measures :

(i) Sufficient practice should be given to the students to convert a given quadratic polynomial as sum or difference of two perfect squares.

(ii) The students should be advised to read the question thoroughly and then decide which standard form of integral is to be used.

(iii) While writing the result involving logs, the expression should be enclosed within modulus sign as $\log \left| x + \sqrt{x^2 + a^2} \right|$

Q. No. 12. Evaluate :

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x}\, dx$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	knowledge	Integration (Definite integral) use of the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Expected Answer :

$$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} = \int_0^{\pi/2} \frac{\sin(\pi/2 - x) - \cos(\pi/2 - x)}{1 + \sin(\pi/2 - x)\cos(\pi/2 - x)}\, dx$$

$$= \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x}\, dx = -I \quad \left(\text{Using } \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$\therefore\ 2I = 0$

or, $I = 0$

Performance level of student s :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
17	18	05	10	–	50	1.36

General Remarks :

83% of the candidates attempted the question. Out of those who attempted, 60% did it correctly. 20% candidates committed conceptual errors while 13% could not solve the question correctly because of computational errors.

Errors committed with examples :

(i) Some candidates tried to solve the question by substitution method and thus arrived at incorrect solution.

(ii) Wrong use of Trigonometrical results such as

$$\sin\left(\frac{\pi}{2}-\theta\right)=\sin\theta,\quad \cos\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

Probable causes :

(i) Candidates are not in a position to recall standard Trigonometric results e.g.

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta,\quad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta \text{ etc.}$$

(ii) Lack of knowledge of application of the property

$$\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$$

Suggested Remedial Measures :

(i) Students are advised to remember standard trigonometric results and sufficient practice should be given to use the formulae properly.

(ii) Sufficient practice in solving problems with the help of properties of definite integrals be given.

(iii) Properties of definite integrals should be given alongwith their proofs.

Q. No. 13. Solve the differential equation

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Separating the variables of a Differential Equation

Expected Answer :

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

separating variables, we get

$$\frac{dy}{\sqrt{1-y^2}}-\frac{-dx}{\sqrt{1-x^2}}$$

Integrating $\int\frac{dy}{1-y^2}=-\int\frac{dx}{\sqrt{1-x^2}}$

$\Rightarrow \sin^{-1}y=-\sin^{-1}x+C$

$\Rightarrow \sin^{-1}y+\sin^{-1}x=C$

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
11	12	4	19	4	50	1.43

General Remarks :

The question has been attempted by as many as 89% candidates from the sample. 39% candidates committed computational errors.

Errors committed with examples :

(i) Some of the candidates considered it as homogenous equation of second degree

(ii) $\int(1-y^2)^{\frac{-1}{2}}\ dy=-\frac{(1-y^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$

(iii) $\sin^{-1}y+\sin^{-1}x=\sin^{-1}C$

$\Rightarrow y+x=c$

(iv) $\int\frac{dy}{\sqrt{1-y^2}}=2y\ \sin^{-1}\ y$

(v) $\frac{dy}{\sqrt{1-y^2}} = -\frac{1}{\sqrt{1-x^2}} dx$

$\frac{dy}{dx} = -\frac{\sin^{-1} x}{\sin^{-1} y}$

(vi) Constant of integration is not written.

Probable causes :

(i) Lack of practice in discriminating different types of differential equations.

(ii) Lack of knowledge of trigonometric functions.

(iii) Results of standard integrals are not known.

Suggested Remedial Measures :

1. Sufficient practice should be given to the students in discriminating different types of differential equations.

2. Difference between $\sin^{-1}\theta$ and $\frac{1}{\sin\theta}$ should be explained to the students clearly.

3. Significance of constant of integration in questions of differential equations should be emphasised upon.

Q. No. 14. Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Probability-Addition Formula and negation

Expected Answer :

Total no. of possible outcomes = 36

unfavourable outcomes

$= \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (4,6), (6,4)\}$

$\Rightarrow$ No of favourable outcomes = 36 – 8 = 28

Required Prob. = $\frac{28}{36} = \frac{7}{9}$

OR

Required Prob. = 1 - P (either a doublet or a total of 10 appears).

$= 1 - \{P\,(\text{doublet}) + P\,(\text{a total of } 10) - P\,(\text{a doublet \& a total of } 10)\}$

$= 1 - \left\{\frac{6}{36} + \frac{3}{36} - \frac{1}{36}\right\}$

$= 1 - \frac{8}{36} = \frac{28}{36} = \frac{7}{9}$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
16	28	17	14	02	23	0.84

General Remarks :

44% of the candidates either did not attempt this question or secured zero mark. Only 23% of the sample could solve the problem correctly. 61% candidates committed conceptual errors.

Errors committed with examples :

(i) P(Doublet) = $\frac{6}{36} = \frac{1}{6}$

P(Total 10) = $\frac{3}{36} = \frac{1}{12}$

P(Neither a doublet nor 10) = $1 - \frac{1}{6} \times \frac{1}{12} = \frac{71}{72}$

(ii) P(No doublet) = $1 - \frac{1}{6} = \frac{5}{6}$

P(Not total 10) = $1 - \frac{1}{12} = \frac{11}{12}$

P(Neither a doublet nor 10) = $\frac{5}{6} \times \frac{11}{12} = \frac{55}{72}$

(iii) $P(\text{Doublet}) = \frac{1}{6}$

$P(\text{Total } 10) = \frac{1}{12}$

$P(\text{Both}) = \frac{1}{6} + \frac{1}{12} = \frac{3}{12}$, $P(\text{Not both}) = 1 - \frac{1}{4} = \frac{3}{4}$

Probable causes :

(i) Candidates failed to determine an event which is a doublet having a total of 10. i.e. (5,5)

(ii) Many of the candidates considered the two events as mutually exclusive events.

Suggested Remedial Measures :

(i) Addition and Multiplication Theorems on probability should be explained clearly.

(ii) The candidates must have the clarity about mutually exclusive events.

(iii) The candidates should be given sufficient practice to write all the favourable events satisfying either of the conditions. Events which are common should be taken only once.

Q. No. 15. Find the regression coefficient of y on x for the following data :

$$\sum x = 24;\ \sum y = 44;\ \sum xy = 306;\ \sum x^2 = 164,\ \sum y^2 = 574;\ n = 4$$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	2	knowledge	regression coefficient of y on x.

Expected Answer :

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$= \frac{306 - \frac{24 \times 44}{4}}{164 - \frac{(24)^2}{4}}$$

$= 2.1$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
10	30	4	6	5	45	1.18

General Remarks :

90% of the candidates from the selected sample attempted this question, out of which about 4% commited conceptual errors. Out of those who attempted, only 50% could complete it correctly securing full marks. Other 50 % committed computational erors, which is not expected of candidates for this stage of education. One third of the candidates who attemped secured zero mark.

Errors Committed with examples :

(i) Wrong formula used

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum y^2 - (\sum y)^2]\,[n\sum x^2 - (\sum x)^2]}} \text{ or } b_{yx} = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}}$$

(ii) Computational mistakes.

(iii) Incomplete solution.

(iv) Calculation of correlation co-efficient in place of regression co-efficient

Probable Causes :

(i) Some of the candidates appear to be confused in writing the formula for b_{yx} and b_{yx}. Many of them have committed computational mistakes either due to lack of practice or due to lack of time.

(ii) Others confused it with correlation coefficient and calculated the same.

Suggested Remedial Measures :

(i) While doing regression lines, students should be made clear about the difference between two regression coefficients and the difference in their formulas

(ii) Sufficient practice may be given to the students to help them eliminate the computational errors and they should be asked to solve the question till they get the final result.

Q. No. 16. Using the properties of determinants, prove that

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Use of properties of determinants

Expected Answer :

Using $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$; we get

$$\text{L.H.S} = (b+c)(c+a)\begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

Using $R_2 \to R_2 + R_1$, We get

$$\text{L.H.S} = (b+c)(c+a)(a+b)\begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -1 \\ 1 & 1 & a+b+c \end{vmatrix}$$

Expanding by R_2 we get

$$\text{L.H.S.} = 2(a+b)(b+c)(c+a) = \text{R. H. S.}$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
9	6	2	3	5	6	2	2	–	65	3.24

General Remarks :

91% of the candidates from the selected sample attempted the question of which 70% could complete the question correctly. Others committed either computational or conceptual errors. 6% candidates secured zero mark.

Errors committed with examles :

(i) Operations performed are not indicated correctly. Sometimes operations are performed without any indication.

(ii) Some candidates have performed row operations even when column operations have been indicated

(iii) Many candidates have found the result by expanding the determinant, without using the properties of determinants.

Probable Causes :

Some of the candidates appear to be confused or in a hurry. Consequently, they made computational mistakes and indicated the operations incorrectly. Sometimes, even the properties are not clear to them and they do not use these correctly and prefer to expand the determinant.

Suggested Remedial Measures :

(i) While doing the properties of determinants, it should be made clear that every performed operation should be cleary indicated Equatities should not be used, Arrows should be used in place of equalities.

(ii) Sufficient practice should be given to the students to transform the given higher order determinant to lower order determinant by using properties only.

(iii) Adequate practice should be given to the students to help them elminate computational errors while using properties.

Q. No. 17. A variable plane passes throngh a fixed point (1, 2, 3). Show that the locus of the foot of perpendicular drawn from the origin to this plane is the sphere given by the equation

$$x^2 + y^2 + z^2 - x - 2y - 3z = 0$$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Plane, sphere and locus

Expected Answer :

Let <a,b,c,> be the direction ratios of the normal to the plane passing throug (1,2,3)

$\therefore$ Equation of plane is

$$a(x-1) + b(y-2) + c(z-3) = 0 \ldots\ldots\ldots\ldots\ldots(i)$$

Let (x_1, y_1, z_1) be the foot of perpendicular from origin to the plane.

$$\therefore \; a(x_1 - 1) + b(y_1 - 2) + c(z_1 - 3) = 0$$

D. Rs. of perpendicular from Origin to the plane are $< x_1, y_1, z_1 >$

$$\therefore \; \frac{a}{x_1} = \frac{b}{y_1} = \frac{c}{z_1} = \lambda \Rightarrow a = \lambda x_1, b = \lambda y, c = \lambda z_1$$

$$\Rightarrow \lambda [x_1 (x_1 - 1) + y_1 (y_1 - 2) + z_1 (z_1 - 3)] = 0$$

$$\Leftrightarrow x_1^2 + y_1^2 + z_1^2 - x_1 - 2y_1 - 3z_1 = 0$$

$\Rightarrow$ Equation of locus is $x^2 + y^2 + z^2 - x - 2y - 3z = 0$ Which is the eqn. of sphere.

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
43	33	1	15	2	1	–	1	–	4	0.69

General Remarks :

76% of the candidates in the sample either did not attempt the question or secured zero mark A number of candidates gave incomplete or vague solutions. Only 7% of the candidates could do the question correctly getting full marks.

Errors committed with examples :

(i) Let the Eqn of sphere be $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

and putting $(-1,-2,-3)$ for 2u, 2v 2w & d = 0

we get $x^2+y^2+z^2-x-2y-3z=0$

(ii) Most of the candidates took (–1, –2, 3) as the foot of ⊥ from origin to the plane and then tried to write the equation of plane.

Probable Causes :

(i) Meaning of term 'locus' is not understood properly by majority of candidates.

(ii) Translation of the problem to mathematical equation of plane could not be done correctly by most of the students.

Suggested Remedial Measures :

Taking into consideration the fact that the problems related locus are generally not understood clearly by many of the students, greater emphasis needs to be given to clarify this concept by taking variety of problems.

Q. No. 18. If a, b, c, are the lengths of the sides opposite respectively to the angle A, B and C of a Δ ABC, Using vectors prove that $\cos C=\frac{a^2+b^2+c^2}{2ab}$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Skill & Understanding	Triangle law, scalar product, angle between two vectors

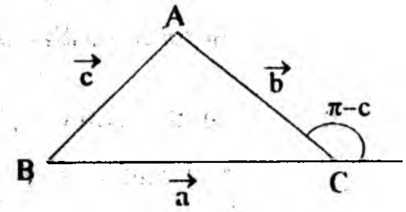
Expected Answer :

Let $\vec{a}, \vec{b}, \vec{c}$ be represented by $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{AB}$ of a ΔABC.

By Δ law of vectors.

$\bar{a}+\bar{b}+\bar{c}=\bar{0}$

$\Rightarrow \bar{c}=-(\bar{a}+\bar{b})$



Squaring, $|\bar{c}|^2=|\bar{a}|^2+|\bar{b}|^2+2\bar{a}.\bar{b}$

$=|\bar{a}|^2+|\bar{b}|^2+2|\bar{a}||\bar{b}|\cos(\pi-c)$

$\Rightarrow c^2=a^2+b^2-2ab\cos C. \quad \because |\bar{a}|=a, |\bar{b}|=b \;\&\; |\bar{c}|=c$

$\Rightarrow \cos C=\dfrac{a^2+b^2-c^2}{2ab}$

Performance level of students :

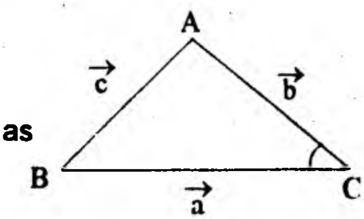
Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
23	8	2	3	–	2	2	15	6	39	3.05

General Remarks :

31% of the candidates from the sample have either not attempted the question or secured zero mark. The response for this question is not satisfactory.

Errors Committed with examples :

(i) Diagram is not properly drawn, It is indicated as



(ii) Concept of angle between the vectors is not clear. e.g. From the diagram, $\vec{a}.\vec{b}$ is taken as $|\bar{a}||\bar{b}|\cos$

Probable Causes :

(i) The concept of angle between the vectors does not seem to be clear,.

(ii) The condition for three vectors to form a triangle is not clear to many.

Suggested Remedial Measures :

(i) The concept of angle between two vectors should be explained clearly. In this case, the angle between a and b is $\pi - c$ and not C.

(ii) The condition for three vectors to form a triangle.

i.e. $\vec{AB} + \vec{BC} + \vec{CA} = 0$ should be emphasised and explained clearly.

Q. No. 19. Two balls are drawn at random from a bag containing 2 white, 3 red , 5 green and 4 black balls, one by one without replacement. Find the probability that both the balls are of different colours.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Probability of dependent events

Expected Answer :

P (both of different colour)

= P(White & Red or White and Green or White and Black or Red & Green or Red & Black or Green & Black)

$$= \frac{{}^2C_1 \cdot {}^3C_1}{{}^{14}C_2} + \frac{{}^2C_1 \cdot {}^5C_1}{{}^{14}C_2} + \frac{{}^2C_1 \cdot {}^4C_1}{{}^{14}C_2} + \frac{{}^3C_1 \cdot {}^5C_1}{{}^{14}C_2} + \frac{{}^3C_1 \cdot {}^4C_1}{{}^{14}C_2} + \frac{{}^5C_1 \cdot {}^4C_1}{{}^{14}C_2}$$

$$= (6+10+8+15+12+20)/91$$

$$= \frac{71}{91}$$

Performance level of Students:

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
23	43	–	6	1	7	3	4	–	13	1.2

General Remarks :

86% of the candidates have either not attempted the question or secured zero mark. Only 13% of the candidates out of the sample could complete the question and secure full marks. Seeing the average score, it seems that the basic concept of probability is not properly understood.

Errors committed with examples :

(i) Probability of selection of 2 white balls $= \frac{2}{14} \times \frac{2}{14}$

(ii) P(White and Red) $= \frac{2_{C_1} + 3_{C_1}}{14_{C_2}}$

Probable Causes :

(i) Candidates do not seem to be clear about the events connected with 'OR' or 'and'.

(ii) It appears that many of them are not able to differentiate between independent events and dependent events.

Suggested remedial measures :

(i) A good number of examples should be discussed to clarify the difference between independent events and dependent events.

(ii) Sufficient practice needs to be provided to find the probability of events connected with the symbol OR or AND.

Q. No. 20. Find the probability distribution of the number of heads in three tosses of a coin.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Probability distribution of a random variable

Expected Ansere :

Here $n = 3,\ p = \frac{1}{2},\ q = \frac{1}{2}$, Let x = Number of heads :

X	0	1	2	3
P(x)	$C(3,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3$	$C(3,1)\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2$	$C(3,2)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$	$C(3,3)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$
=	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Performance Level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
21	18	5	4	1	2	1	6	–	42	2.65

General Remarks :

39% of the candidates in the sample either did not attempt this question or scored zero mark. Only 53% of the candidates who attempted the question could do it correctly.

Errors Committed with examples :

Some of the candidates have written the probability of success as $P = \frac{1}{8}$ instead of $\frac{1}{2}$ which led to erroneous calculations. Most of the errors committed are conceptual in nature.

Probable Causes :

Lack of basic concept of probability and probability distribution of a random variable seems to be the most probable cause for wrong answer.

Suggested remedial Measures :

Adequate number of problems and examples on probability distribution on random variable should be given to the students to clarify this concept.

Q. No. 21. For the function $f(x) = -2x^3 - 9x^2 - 12x + 1$, find the interval(s)

(a) in which f(x) is increasing

(b) in which f(x) is decreasing

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Increasing and decreasing function

Expected Answer :

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$f'(x) = -6x^2 - 18x - 12$$

For increasing and decreasing function, $f'(x) = 0$

$$-6(x^2 + 3x + 2) = 0$$

$$\Rightarrow -6(x+1)(x+2) = 0$$

$$\Rightarrow x = -1, -2$$

Interval	(x+1)	(x+2)	$f'(x) = -6(x+2)(x+1)$	Nature of function
$x < -2$	–ve	–ve	–ve	Decreasing
$-2 < x < -1$	–ve	+ve	+ve	Increasing
$x > -1$	+ve	+ve	–ve	Decreasing

f(x) is decreasing in $x < -2$, $x > -1$ and f(x) is increasing in $-2 < x < -1$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
7	5	1	7	3	27	14	7	2	27	2.55

General Remarks :

The question has been attempted by majority of candidates, though only 29% of those who attempted could do it correctly. Majority of candidates could not understand the question because of negative sign with the function f(x)

Errors committed with examples :

(i) $f'(x) = -6(x+1)\ (x+2)$

- ve sign of the derivative was not considered for determining the nature of the function.

(ii) $f'(x) = -6x^2 - 18x - 12$

$= 6(x-1)\,(x-2)$

$\therefore\ f'(x) = 0 \Rightarrow x = 1, 2$

(iii) Inequation could not be solved by majority of candidates.

(iv) Computational errors in factorization.

Probable Causes :

(i) Lack of practice in simple factorization of a polynomial.

(ii) Lack of knowledge in solving inequation.

(iii) $f'(x)$ is not considered in totality. Nature of (x+1) and (x+2) is considered for checking the sign of the derivative.

Suggested Remedial Measures :

(i) Sufficient practice should be given in finding the factors of polynomial of the type $ax^2 + bx + c$

(ii) Concept of inequations should be made clear by explaining the meaning of a b > 0 and a b < 0

(iii) The students should be made to understand clearly that for the nature of function, it is the behaviour of f'(x) which is to be considered and not simply the factors of $f'(x)$.

Q. No. 22. Find from first principle the derivative of $\sqrt{\operatorname{cosec} x}$ w. r. t. x.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Differentiation by first principle

Expected answer :

$$y = \sqrt{\operatorname{cosec} x}$$

$$\Delta y = \sqrt{\operatorname{cosec}(x+\Delta x)} - \sqrt{\operatorname{cosec} x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{\operatorname{cosec}(x+\Delta x)} - \sqrt{\operatorname{cosec} x}}{\Delta x} \times \frac{\sqrt{\operatorname{cosec}(x+\Delta x)} + \sqrt{\operatorname{cosec} x}}{\sqrt{\operatorname{cosec}(x+\Delta x)} + \sqrt{\operatorname{cosec} x}}$$

$$= \frac{-[\sin(x+\Delta x) - \sin x]}{\sin(x+\Delta x)\,.\,\sin x\,.\,\Delta x} \times \frac{1}{\sqrt{\operatorname{cosec}(x+\Delta x)} + \sqrt{\operatorname{cosec} x}}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\cos\left[x+\frac{\Delta x}{2}\right]\sin\frac{\Delta x}{2}}{\sin(x+\Delta x)\sin x\,\frac{\Delta x}{1}} \cdot \frac{1}{\sqrt{\operatorname{cosec}(x+\Delta x)} + \sqrt{\operatorname{cosec} x}}$$

$$\therefore \frac{dy}{dx} = -\frac{\operatorname{cosec} x \cot x}{2\sqrt{\operatorname{cosec} x}}$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
7	7	5	12	6	12	4	6	4	37	2.55

General Remarks :

The question has been attempted by a vast majority but 56% candidates committed computational and conceptual errors which is very alarming.

Errors committed with examples :

(i) Rationalisation has not been done

(ii) Candidates have used C,D formula to simplify $\sqrt{\operatorname{cosec}(x+\Delta x)}-\sqrt{\operatorname{cosec} x}$

(iii) $\sqrt{\operatorname{cosec}(x+\Delta x)}-\sqrt{\operatorname{cosec} x}=\sqrt{\operatorname{cosec}(x+\Delta x-x)}$

(iv) $\operatorname{cosec}(x+\Delta x)=\operatorname{cosec} x+\operatorname{cosec}\Delta x$

(v) $\sqrt{\operatorname{cosec}(x+\Delta x)}-\sqrt{\operatorname{cosec} x}=\sqrt{\operatorname{cosec}(x+\Delta x)-\operatorname{cosec} x}$

(vi) $\sin C-\sin D=2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(vii) Taking derivative without taking limit as $\Delta x \to 0$

Probable Causes :

(i) Basic concept of radical sign is not clear.

(ii) Lack of practice in rationalisation.

(iii) Lack of knowledge of CD formula.

Suggested Remedial Measures :

(i) Some of the basic concepts of radical sign should be made clear e.g. $\sqrt{a}\pm\sqrt{b}\neq\sqrt{a\pm b}$

(ii) Sufficient practice on rationalisation should be given

(iii) Knowledge of trigonometry should be strengthened.

(iv) The concept of limits also require greater clarification. A lot of practice is required in this area.

Q. No. 23. Find the value of $\int_0^2 (3x^2-2)\,dx$ as limit of sums.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Integration of functions as limit of sums

Expected answer :

$a = 0, \; b = 2$

$h = \frac{2-0}{n} = \frac{2}{n}$

$= \int_0^2 (3x^2 - 2)\,dx = \underset{n \to \infty}{Lt} \frac{2}{n}\left[f(o) + f(h) + f(2h) + \ldots + f(\overline{n-1}\;h)\right]$

$= \underset{n \to \infty}{Lt} \frac{2}{n}\left[(-2 + (3h^2 - 2) + (12h^2 - 2) + \ldots + \left\{3(n-1)^2 h^2 - 2\right\}\right]$

$= \underset{n \to \infty}{Lt} \frac{2}{n}\left[-2n + \frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$

$= 4$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
09	05	03	03	07	17	03	10	05	38	2.70

General Remarks :

As many as 81% candidates of the sample attempted the question though only 42% of them could do it correctly. The rest of candidates could not complete the solution because of computational errors. As many as 20% of the candidates committed conceptual errors.

Errors Committed with examples :

(i) $h = \frac{b-a}{n}$ is written as $h = \frac{0-2}{n} = \frac{-2}{n}$

(ii) $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ has not been used correctly. Some candidates have used $\sum (n-1)^2 = \frac{n(n-1)(2n+1)}{6}$

(iii) The concept of limits that when $n \to 0, \; h \to \infty$ is not clear.

(iv) Many of the candidates are not aware about the correct formula for limits of sums.

Probable Causes:

(i) Inadequate knowledge about correct formula for limit of the sum.

(ii) Lack of practice in applying standard results such as $\sum n, \sum n^2, \sum n^3$ etc.

Suggested remedial measures:

(i) Sufficient practice needs to be given to find `h` correctly.

(ii) The students should be helped to learn the formula for evaluating the limit of sum correctly.

(iii) The students need to be given sufficient practice for finding integral as limit of sums of the type $\int_a^b e^x\,dx$, $\int_p^q (ax+b)\,dx$, $\int_a^b$ sin x dx etc.

Q. No. 24. Evaluate : $\int \frac{3}{(1-x)(1+x^2)}\,dx$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	integration using partial fractions

Expected Answer :

$$= \frac{3}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$A = \frac{3}{2},\ B = \frac{3}{2},\ C = 3/2$$

$$\therefore \int \frac{3}{(1-x)(1+x^2)}\,dx = \frac{3}{2}\int \frac{1}{1-x}\,dx + \frac{3}{2}\int \frac{x+1}{1+x^2}\,dx$$

$$= \frac{3}{2}\int \frac{1}{1-x}\,dx + \frac{3}{2}\int \frac{x}{1+x^2}\,dx + \frac{3}{2}\int \frac{1}{1+x^2\,dx}$$

$$= \frac{-3}{2}\log|1-x| + \frac{3}{4}\log(1+x^2) + \frac{3}{2}\tan^{-1}x + c$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
13	17	–	09	05	11	03	05	11	26	2.34

General remarks :

87% of the candidates in the sample attempted this question. Out of those who attempted, 30% did it correctly, as many as 32% candidates committed computational errors while 28% candidates committed conceptual errors.

Errors committed with examples :

(i) $\frac{3}{(1-x)(1+x^2)}$ is written as $\frac{A}{1-x}+\frac{B}{1+x^2}$. while others wrote

$$\frac{3}{(1-x)(1+x^2)}=\frac{A}{1-x}+\frac{B}{(1+x^2)}+\frac{C}{(1+x^2)}$$

(ii) Computational errors in calculating the value of A, B, C

(iii) Failure to intergate $\int\frac{x+1}{1+x^2}dx$ correctly.

Probable causes :

(i) Lack of knowledge of methods of writing the given expression into partial fractions.

(ii) Lack of computational skill in determining the value of constants in partial fractions.

(iii) Lack of basic concept of integration for questions of the type $\int\frac{x+1}{x^2+1}dx$

Suggested Remedial Measures :

(i) Students should be given sufficient practice in splitting a given fraction (different topics) into partial fractions.

(ii) Students should be helped to be more careful and avoid unnecessary computational errors.

(iii) Sufficient practice is required in evaluating different type of integrals especially of the type $\int \frac{ax+b}{x^2+1} dx$.

Q. No. 25. Draw a rough sketch and find the area of the region bounded by the two parabolas $y^2 = 4x$ and $x^2 = 4y$ by using methods of intergration.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Integration (To find area under curves)

Expected Answer :

x-Coordinate of the point of intersection of the given curves = 4

$\therefore$ Area bounded by the curves $= \int_6^4 (2\sqrt{x} - \frac{x^2}{4})\, dx$

$$= \left| \left(\frac{2 \cdot x^{3/2}}{3/2} - \frac{x^3}{12} \right) \right|_0^4$$

$$= \frac{16}{3} \text{ sq. units}$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
24	07	04	10	02	09	02	12	02	30	2.63

General Remarks :

76% of the candidates attempted this question. Out of those who attempted, 39% did it correctly, 30% candidates made conceptual errors while as many as 24% could not get full marks because of computational errors.

Errors committed with examples :

(i) Graphs of the curves are not drawn properly.

(ii) Shading of the required region is not done properly

(iii) Area of the region is calculated as $\int_0^4 \left(\frac{x^2}{4} - 2\sqrt{x}\right)dx$ which comes to be negative.

(iv) Computational errors are committed while finding the point of intersection of the curves.

(v) Units of area are not written.

Probable causes :

(i) Inadequate knowledge of plotting graphs of curves.

(ii) Poor computational skill in calculating the point of intersection of two curves.

Suggested Remedial Measures :

(i) A lot of practice is desired in drawing rough sketches of graphs of parabolas of the type $y^2 = ax, x^2 = by$ etc.

(ii) Students should be given sufficient practice in finding the point of intersection of the curves and the x coordinates in particular

(iii) Students should be asked to express the answer with proper units.

(iv) It should be clarified that area is always positive and it should not be expressed as negative.

Q. No. 26. If $A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$, Find A^{-1}

Using A^{-1}, solve the following system of linear equations.

$$4x - 5y - 11z = 12$$
$$x - 3y + z = 1$$
$$2x + 3y - 7z = 2$$

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	To find inverse and hence solve system of linear equations

Expected Answer :

$$|A| = -72$$

$$\text{Adj }(A) = \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{-1}{72}\begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

Using $x = A^{-1}B$

$$x = -1;\ y = -1,\ z = -1$$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
5	–	–	1	1	11	9	15	3	14	10	4	6	21	4.0

General remarks :

90% of the candidates from the selected sample attempted this question out of which about 24% commited conceptual errors. Only 22% of those who attempted, could do this question correctly getting 6 marks. About 75% of the selected sample followed the desired method whereas about 13% could not complete the solution.

Errors Committed with examples :

(i) Notation of determinant has been used in place of matrix notation by a large number of students.

(ii) Many students committed computational mistakes while finding the value of the detrminant and adjoint. In case of adjoint, +ve and -ve signs were not taken care of.

(iii) Linear equations are not solved after finding the inverse.

(iv) Some candidates solved these equations algebriacally.

(v) Even after finding the inverse correctly, some used cramer's rule for solving the linear equations instead of the inverse method.

Probable Causes :

(i) Some of the candidates appear to be confused with the notation of matrix and a determinant.

(ii) Many of them made computational mistakes either due to lack of practice or due to being in a hurry.

(iii) Some of the candidates have not understood the meaning of the phrase "and hence...." They have used cramer's Rule and solved the equation.

Suggested Remedial Measures :

(i) The students should be helped to understand the difference between notation for matrices and determinants.

(ii) Sufficient practice/should be given to students to find the adjoint of matrices and they should be asked to take care of the positive and negative signs while calculating it.

(iii) They should be made to understand the method of finding the solution of linear equations by inverse method as desired.

(iv) They should be asked to read the question carefully before attempting it.

Q. No. 27. Define the line of shortest distance between two skew lines. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by

$$\bar{r} = (2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} - \hat{j})$$

$$\bar{r} = (4\hat{i} + 3\hat{k}) + \mu(3\hat{i} + \hat{j} + \hat{k})$$

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Understanding	Skew lines/Shortest Distance/Equation of line.

Expected Answer :

Def: The line perpendicular to both the skew lines is the line of shortest distance

Points P $(2\lambda, 2-\lambda, -3)$, $Q(4+3\mu, \mu, 3+\mu)$ lie on the lines L_1 and L_2

DR's of $\vec{PQ} < 3\mu - 2\lambda + 4, \mu + \lambda - 2, \mu + 6 >$. DR's of L_1 and L_2 are $<2, -1, 0>$ & $<3, 1, 1>$ resp.

$\vec{PQ} \perp \vec{L_1} \Rightarrow 5\mu - 5\lambda + 10 = 0$ ....(1) (Using, $a_1a_2 + b_1b_2 + c_1c_2=0$)

$\vec{PQ} \perp \vec{L_2} \Rightarrow 11\mu - 5\lambda + 16 = 0$..........(2)

Solving (1) and (2), we get $\lambda = 1$, $\mu = -1$

$\Rightarrow P(2, 1, -3)$ and $Q(1, -1, 2)$

Also Shortest Distance = $\left|\vec{PQ}\right| = \sqrt{30}$

Vector Equation of line is

$\vec{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
4	7	–	10	5	17	6	26	3	12	3	2	1	4	2.67

General Remarks :

Though 96% of the candidates tried to attempt the question, just 4% of them could give complete solution and scored full marks Average score is 2.67 which is very discouraging. Very few students defined the line of shortest distance between two skew lines The method of finding the equation of the line of shortest distance was not understood by many of them.

Errors Committed with examples :

(i) Instead of defining the line of shortest distance between two skew lines, many of the students defined skewlines.

(ii) Computational error in finding the shortest distance between two lines by a particular formula i.e.

$$\text{S.D.} = \frac{(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)}{\left|\bar{b}_1 \times \bar{b}_2\right|}$$

Probable Causes :

(i) The concept that the line of shortest distance is perpendicular to both skew lines is not clear to majority of candidates.

(ii) Concept of perpendicularity of lines is not understood by many.

(iii) Equation of line passing through two given points is not clear to a significant number.

Suggested Remedial Measures :

(i) Greater emphasis needs to be laid on explaining the concepts of skew lines and shortest distance between two skew lines.

(ii) Sufficient practice needs to be given to the students to find the equation of the line of shortest distance between two skew lines.

Q. No. 28. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour, when will it have lost 95% moisture, weather conditions remaining the same.

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	Application of differential equaltions.

Expected Answer :

Let M be the moisture content present at any time t.

$$\Rightarrow \frac{dM}{dt} \alpha M \quad \text{or} \quad \frac{dM}{dt} = KM$$

$$\Rightarrow \frac{dM}{dt} = K.dt$$

Inlegrating $\int \frac{dM}{M} = K \int dt \Rightarrow \log M = Kt + C$ ....(1)

When $t = 0$, Let $M = M_0 \Rightarrow \log M_0 = C$

(1) becomes $\log M = Kt + \log M_0$

at $t = 1$, $M = \frac{Mo}{2}$, $\log \frac{Mo}{2} = K + \log Mo \Rightarrow K = -\log 2$

When $M = \frac{5}{100} Mo$, $\log M = -t \log 2 + \log Mo$

$$\Rightarrow \log \frac{5}{100} Mo = -t \log 2 + \log Mo$$

$$-t \log 2 = -\log 20$$

$$\Rightarrow t = \log 20 / \log 2.$$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1½	2	2½	3	3½	4	4½	5	5½	6	
51	16	2	9	–	2	3	6	–	4	–	1	1	5	1.95

General Remarks :

The question has been very poorly attempted. 51% candidates from the sample did not attempt the question. Only 5% candidates from the sample could attempt it correctly. This topic needs special attention of both teachers and students.

Errors Committed with examples :

(i) Many candidates failed to translate word problem into differential equation.

(ii) Instead of taking 5% moisture content at t = 1, it has been taken as 95%.

(iii) Given conditions are not used properly for calculating constants of integration and constant of proportionality.

(iv) Constant of integration is not taken into consideration.

(v) log 20 / log 2 = log 20 – log 2.

Probable Causes :

(i) Inadequate mastery in transtation of word problems into mathematical equations.

(ii) Lack of knowledge in solving differential equations under given conditions.

(iii) Lack of knowledge of basic concepts of logarithms.

Suggested Remedial Measures :

(i) Sufficient practice needs to be given to students in translating word problems into mathematical equations.

(ii) More practice should be given to the students in solving differential equations and finding the value of constant of integration under given conditions.

Q. No. 29. A rectangle is inscribed in a semi-circle of radius r with one of its sides on the diameter of the semi circle. Find the dimensions of the rectangle so that its area is maximum. Find also the area.

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	Application of derivatives

Expected Answer :

$$x^2 + y^2 = r^2$$

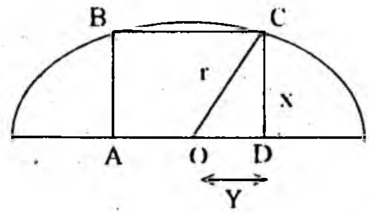
Area of the rectangle = 2xy

$$A = 2x\sqrt{r^2 - x^2}$$

Let $S = A^2 = 4x^2(r^2 - x^2)$

$$S = 4x^2r^2 - 4x^4$$

$$\Rightarrow \frac{ds}{dx} = 8r^2x - 16x^3$$



Putting $\frac{ds}{dx} = 0 \Rightarrow 8r^2x - 16x^3 = 0$

$$x^2 = \frac{1}{2}r^2 \Rightarrow x = \frac{1}{\sqrt{2}}r$$

$$\frac{d^2s}{dx^2} = 8r^2 - 48x^2 = 8r^2 - 48\left(\frac{1}{\sqrt{2}}r\right)^2 = 8r^2 - \frac{48}{2}r^2 = -16r^2$$

$\frac{d^2s}{dx^2} < 0$, $\therefore$ Area is maximum

$$x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 = r^2 - \frac{1}{2}r^2$$

$$\Rightarrow y = \frac{1}{\sqrt{2}}r$$

Length of rectangle $\frac{2}{\sqrt{2}}r = \sqrt{2}\,r$

Max. Area $= \frac{1}{\sqrt{2}}r \times \sqrt{2} = r^2$

Performance level of students:

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
41	27	5	11	2	1	–	2	–	3	1	3	–	4	1.35

General Remarks:

The question has been very poorly attempted. Only 4% candidates could attempt it correctly. 41% of the candidates did not attempt the question at all.

Errors Committed with examples :

(i) Translation of the statement into mathematical equation is not done properly.

(ii) Diagram is not drawn correctly.

(iii) Parameter is maximised in place of area.

(iv) Instead of taking area of the rectangle, many of the candidates have taken the area of the whole figure.

(vi) For maximum area, the second derivative test is not carried out.

(vii) Area of the rectangle is not determined.

Probable Causes :

(i) Many candidates failed to translate word problem into mathematical equation.

(ii) The concept of maxima and minima is not clear to a significant number of candidates They seem to have done the question mechanically.

Suggested Remedial Measures :

(i) Sufficient practice should be given to the students to translate word problems into mathematical equations.

(ii) The concept of maxima and minima needs to be made more clear to the students.

(iii) The students should be advised to read the question thorughly before starting to solve the same.

Q. No. 30.

Find karl pearson's coefficient of correlation between x and y for the following data :

x :	16	18	21	20	22	26	27	15
y :	22	25	24	26	25	30	33	14

Also give the interpretation of the coefficient of correlation thus obtained.

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Understanding	Coefficient of correlation & its interprelation

Expected Answer :

$$p(x, y) = \frac{\sum dxdy - \frac{\sum dx \sum dy}{n}}{\sqrt{\sum dx^2 - \frac{(\sum dx)^2}{n}}\sqrt{\sum dy^2 - \frac{(\sum dy)^2}{n}}}$$

With $\bar{x} = 20$ & $\bar{y} = 25$

$\sum dx = 5 \quad \sum dy = -1 \quad \sum dxdy = 152 \quad \sum dx^2 = 135 \quad \sum dy^2 = 221$

$= 0.89$

Interpretation :

High degree of positive correlation i.e. x increasas or decreases with increase or decrease in Y and vice versa.

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
2	1	–	3	6	6	5	20	10	26	16	5	–	–	3.40

General Remarks :

98% of the candidates from the selected sample attempted this question out of which 35% used wrong formula. Not even a single candidate from the selected sample could do this question correctly. About 66% of the selected sample left the question incomplete and number of them made computational errors.

Errors Committed with examples :

(i) Formula wrongly used as

$$p(x, y) = \frac{\sum dxdy - \frac{\sum dx \sum dy}{n}}{\left[\sum dx^2 - \frac{(\sum dx)^2}{n}\right]\left[\sum dy^2 - \frac{(\sum dy)^2}{n}\right]}$$

(ii) More than 60% of the students have left the question incomplete. Many of them left the question without calculating the final answer.

(iii) Majority of them have committed computatinal errors.

(iv) p (x, y) has been calculated to be more than 1.

Probable Causes :

Majority of the candidates do not seem to remember correct formula. The fact that many of them could not interpret the result is an indicator that they do not understand the physical significance of coefficent of correlation.

Suggested Remedial Measures :

(i) Interpretation of the result should be made clear at the time of teaching the topic in the class.

(ii) Sufficient practice should be given to the students to do the calculations with the help of logarithm table.

(iii) The students should be asked to learn the formula carefully.

(iv) It should be made clear to the students that the value of p lies between -1 and +1.

TOPIC WISE STATISTICAL SUMMARY OF STUDENTS' PERFORMANCE.

MATRICES **N=100**

Question Number	No. of candidates attempting the Question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
1	89	28	21	9	5	26	0.80	40.00
2	78	12	9	12	10	35	1.31	65.50
16	91	6	5	11	4	65	3.24	81.00
26	95	0	2	35	27	31	4.00	66.67

In the unit on Matrices and Determinants, the overall performance of candidates has been satisfactory. Leaving aside Q No. 2 (which has been attempted by 78% candidates), each question has been attempted by about 90% candidates. The mean percentage score on this unit ranges from 40% in Q.No.1. to 81% in Q.No.16. About 65% of those who attempted Q No. 1 scored marks below 50% whereas more than 70% of those who attempted the question secured more than 75% marks in QNo. 16. The above table indicates that the performance of the candidates is poorest in Q. No. 16 and best in Q.No.16.

VECTORS AND 3-DIMENSIONAL GEOMETRY

N = 100

Question Number	No. of candidates attempting the Question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
3	89	17	2	6	1	6.3	1.51	75.50
4	56	31	4	5	1	15	0.70	35.00
17	57	33	16	3	1	4	0.69	17.25
18	77	8	5	2	17	45	3.05	76.25
27	96	7	15	49	18	7	2.67	44.50

In the unit on vectors and 3D Geometry, the best performance has been in Q.No. 18 (mean percentage score of 76.25) followed by Q. No. 3. (mean score 75.50) The worst performance has been in Q.No. 17 with a mean percentage score of 17.25

Q. No.s 4 and 17 have been found to be difficult by the candidates with mean percentage scores of 35.00 and 17.25 respectively.

DIFFERENTIAL CALCULUS

N = 100

Question Number	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
5	98	5	3	22	14	54	1.56	78.00
6	85	42	4	9	1	29	0.84	42.00
7	83	34	6	10	2	31	0.94	47.00
21	93	5	8	30	21	29	2.55	63.75
22	93	7	17	18	10	41	2.55	63.75
29	59	27	18	3	4	7	1.35	22.50

In the unit on differential calculus, the best performance has been on Q. No. 5 (with a mean percentage score of 78%) followed by mean percentage scores of 63.75 each on Q. Nos. 21 and 22.

In Q. Nos. 5 and 22, about 60% of the candidates secured 75% or more marks. In Q. No. 6, more than 60% of the candidates secured less than 50% marks.

Q. No. 29 has been attempted by least number of candidates (59%) and more than 70% of them secured 50% or less marks.

INTEGRAL CALCULUS N=100

Question Number	No. of Candidates attempting the Question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
8	91	14	–	1	2	74	1.82	91.00
9	79	34	5	4	2	34	0.98	49.00
10	88	27	6	17	5	33	1.06	53.00
11	83	11	2	14	5	51	1.50	75.00
12	83	18	5	10	–	50	1.36	68.00
23	91	5	6	24	13	43	2.70	62.50
24	87	17	9	16	8	37	2.34	58.50
25	76	7	12	11	14	32	2.63	65.75

In the topic on integral calculus, all the questions have been attempted by more than 75% candidates, the largest (91%) being in Q. No. 8 and 23, and the least (76%) being on Q.No. 25, The least mean score 49% has been in Q. No. 9, followed by 53% in Q.No. 10 and 58.50% in Q. No. 24.

More than 80% of the candidates who attempted the question, secured 75% or more marks in Q. No. 8, The corresponding percentage for Q. No. 11 and Q.No. 12 is 61% and 60% respectively.

DIFFERENTIAL EQUATIONS

N=100

Question Number	No. of candidates attempting the Question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
13	89	12	4	19	4	50	1.43	71.50
28	49	16	11	11	4	7	1.95	32.50

Q. No. 13 in this unit has been attempted by about 90% candidates whereas the second question (N0.28) has been attempted by 49% candidates only.

In Q. No. 13, about 55% of the candidates got more than 75% or more marks. The corresponding figure for Q. No. 28 has been about 15% . This means that Q. No., 28 has been found to be much more difficult by the candidates as against Q. No. 13.

CORRELATION & REGRESSION

N = 100

Question Number	No. of candidates attempting the question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
15	90	30	4	6	5	45	1.18	59.00
30	98	1	9	31	52	5	3.40	56.67

Both the questions (No. 15 & 30) have been observed to be popular by the candidates and 90% and 98% candidates have attempted these, with mean percentage scores of 59 and 56.67 respectively.

About 33% of the candidates who attempted Q. No. 15 secured zero mark whereas the corresponding figure for the other question (No 30) is about 1%

About 50% of those who attempted Q.No. 15, secured 75% or more marks..

PROBABILITY

N = 100

Question Number	No. of candidates attemptnig the question	Number of candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
14	84	28	17	14	2	23	0.86	43.00
19	77	43	6	8	7	13	1.20	30.00
20	79	18	9	3	7	42	2.65	66.25

Although all the three questions have been attempted by more than 75% canddiates, the average (mean) percentage scores have been much below expectations.

About 75% of those who attempted Q. No. 19, secured less than 50% marks. More than 50% of the candidates who attempted secured 75% or more marks.

OVERALL STATISTICAL SUMMARY FOR CODE 65/1/1

Question Number	No.of cand. attem-pting the Q.	Number of candidates getting marks											Mean	Mean Score	Mean % Score
		0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
1	89	28	21	9	5	26								0.80	40.00
2	78	12	9	12	10	35								1.31	65.50
3	89	17	2	6	1	63								1.51	75.50
4	56	31	4	5	1	15								0.70	35.00
5	98	5	3	22	14	54								1.56	78.00
6	85	42.	4	9	1	29								0.84	42.00
7	83	34	6	10	2	31								0.94	47.00
8	91	14	–	1	2	74								1.82	91.00
9	79	34	5	4	2	34								0.98	49.00
10	88	27	6	17	5	33								1.06	53.00

11	83	11	02	14	5	51									1.50	75.00
12	83	18	5	10	–	50									1.36	68.00
13	89	12	4	19	4	50									1.43	71.50
14	84	28	17	14	2	23									0.86	43.00
15	90	30	4	6	5	45									1.18	59.00
16	91	6	2	3	5	6	2	2	–	65					3.24	81.00
17	57	33	1	15	2	1	–	1	–	4					0.69	17.25
18	77	8	2	3	–	2	2	15	6	39					3.05	76.25
19	77	43	–	6	1	7	3	4	–	13					1.20	30.00
20	79	18	5	4	1	2	1	6	–	42					2.65	66.25
21	93	5	1	7	3	27	14	7	2	27					2.55	63.75
22	93	7	5	12	6	12	4	6	4	37					2.55	63.75
23	91	5	3	3	7	17	3	10	5	38					2.70	62.50
24	87	17	–	9	5	11	3	5	11	26					2.34	58.50
25	76	7	2	10	2	9	2	12	2	30					2.63	65.75
26	95	–	–	1	1	11	9	15	3	14	10	4	6	21	4.00	66.67
27	96	7	–	10	5	17	6	26	3	12	3	2	1	4	2.67	44.50
28	49	16	2	9	–	2	3	6	–	4	–	1	1	5	1.95	32.50
29	59	27	5	11	2	1	–	2	–	3	1	3	–	4	1.35	22.50
30	98	1	–	3	6	6	5	20	10	26	16	5	–	–	3.40	56.67

OUTSIDE DELHI REGIONS

Q. No. 1. By using elementary row transformations, find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Elementary row transformations on a matrix

Expected Answer :

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$$

$R_2 \to \frac{1}{5} R_2$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A$$

$R_1 \to R_1 + R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{bmatrix} A \qquad \therefore A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
7	28	13	13	6	33	1.02

General Remarks :

93% of the candidates from the selected sample attempted the question, out of which about 32% committed conceptual errors. Only 33% candidates could do the question correctly getting full marks. About 30% candidates from the sample secured zero mark. About 50% of the students followed the desired method in proper form but 10% could not complete it.

Errors Committed with examples :

(i) Symbol of a matrix has not been used by some of the candidates. Instead they have used the notation of a determinant.

(ii) Instead of using row transformations, some of the students have used column transformations.

(iii) Transformations have been indicated wrongly. For example, some candidates have used $R_2 \to R_2 + R_1$ but have indicated $R_2 \to C_2 + C_1$, etc.

(iv) Some candidates have committed computational errors for the matrix using $R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ instead of $\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$ etc.

(v) Some candidates have missed writing "A" in the first step.

Probable Causes :

Some of the candidates appear to be confused with the notations of a determinant and a matrix They got mixed up with row/column transformation and thereby made column transformation in place of row transformations.

Suggestec Remedial Measures :

(i) The students should be helped to understand the difference betweeen row transformation and column transformation. They should be clearly told that only row transformation are to be used for finding the inverse of the given matrix as desired in the question.

(ii) Use of notation should be clearly explained to the students and its way of writing should also be explained properly. The students need to be given sufficient practice so that they can use correct notations and do not commit computational errors.

Q. No. 2. Compute the adjoint of the matrix A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify that A. (Adj A) = $|A|$ I.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Computation of Adjoint and $\lvert A \rvert$

Expected Answer :

$$|A| = -11$$

$$\text{Adj. } A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A.\ (\text{Adj } A) = \begin{pmatrix} -11 & 0 \\ 0 & -11 \end{pmatrix}$$

$$= -11 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= |A|\ I$$

Performance level of students :

Number of students not attemnting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
2	7	3	10	1	77	1.74

General Remarks :

98% of the candidates out of the selected sample have attempted the question and about 78% of them have done it correctly, showing thereby that such questions are populár amongst students. About 10% candidates have committed computational / conceptual errors and 5% have left the solution incomplete.

Errors Committed with examples :

(i) Symbol of a determinant has been used in place of the symbol for matrix

(ii) Wrong multiplications of matrices leading to incorrect result.

(iii) The value of determinant of the matrix, i.e. (A) has been found wrongly. Some candidates have understood it to be the modulus of the value of det. A and have taken it as 11 in place of - 11.

(iv) The adjoint A has not been calculated correctly.

Probable causes :

(i) The idea of finding co-factor of matrix and then taking its transpose for finding the adjoint of a matrix is not clear to candidates.

(ii) The candidates do not have sufficient practice in handling the multiplication of matrices due to which they commit computational errors.

(iii) |A| means the determinant of the matrix A and not its modulus. This point is not clear to many students.

Suggested remedial Measures :

(i) The students need to be given sufficient practice in handling multiplication of matrices.

(ii) The key to such questions is finding co-factors correctly The students should be given sufficient practice to handle such questions and finding adjoint of a given matrix.

(iii) Meaning of |A| should be clarified with the help of examples.

(iv) Difference between adjoint and co-factors of matrix should be clearly explained to the students.

Q. No. 3. Define $\vec{a} \times \vec{b}$ and prove that $\left|\vec{a} \times \vec{b}\right| = (\bar{a}.\bar{b})$ tan θ where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$,

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Knowledge	Dot product and Cross product of two vectors

Expected Answer :

Def: If $\bar{a}$ and $\bar{b}$ are any two vectors then their vector product $\bar{a} \times \bar{b}$ is defined as $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \sin\theta \, \hat{n}$ where θ is the angle betgween $\bar{a}$ and $\bar{b}$ and $\hat{n}$ is a unit vector perpendicular to both $\bar{a}$ and $\bar{b}$ such that $\bar{a}, \bar{b}$ and $\hat{n}$ form a right handed system.

Since $\left|\bar{a} \times \bar{b}\right| = |\bar{a}||\bar{b}| \text{Sin}\,\theta$

$$= |\bar{a}||\bar{b}| \quad \text{Cos}\,\theta \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}$$

$$= \bar{a}\,.\,\bar{b} \tan\theta \quad \left[\because \bar{a}.\bar{b} = |\bar{a}|\,|\bar{b}| \text{Cos}\,\theta\right]$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
17	11	7	3	17	45	1.47

General Remarks :

83% of the candidates from the sample attempted this question and about 54% of them secured full marks. 46% of those who attempted this question made conceptual errors in defining the vector product.

Errors Committed with examples :

Some of the candidates have written $\bar{a} \times \bar{b}$ as

(i) $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \, \mathrm{Sin}\,\theta$, or

(ii) $a \times b = ab \, \mathrm{Sin}\theta \, \hat{n}$, or

(iii) $a \,.\, b = ab \, \mathrm{Sin}\theta$

in place of $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \, \mathrm{Sin}\theta \, \hat{n}$

Probable Causes :

(i) Difference between a scalar and a vector notation is not clear to many candidates.

(ii) Some candidates do not appear to be aware of the fact that the cross product of two vectors is a vector.

Suggested Remedial Measures :

Sufficient number of examples should be given to make the following concepts clear :

(i) multiplication of a vector by a scalar

(ii) dot product of two vectors

(iii) cross product of two vectors

(iv) difference betweeen a scalar and a vector.

Q. No. 4. Show that the points A,B,C, and D with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ respectively and such that $5\vec{a} - 3\vec{b} + 4\vec{c} - 6\vec{d} = \vec{0}$, are co-planers.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Coplanarity of vectors

Expected Answer :

$$5\vec{a} - 3\vec{b} + 4\vec{c} - 6\vec{d} = \vec{0}$$

$$\Rightarrow 5\vec{a} + 4\vec{c} = 3\vec{b} + 6\vec{d}$$

$$\text{or } \frac{5\vec{a} + 4\vec{c}}{9} = \frac{3\vec{b} + 6\vec{d}}{9} \text{ or } \frac{5\vec{a} + 4\vec{c}}{9} = \frac{\vec{b} + 2\vec{d}}{3}$$

The position vector of a point dividing AC in the ratio 4 : 5 is same as the position vector of a point dividing BD in the ratio 2 : 1

i.e. AC and BD are intersecting lines

$\therefore$ Ac and BD are coplanar [$\because$ intersecting lines are always coplaner]

$\therefore$ A,B,C,D are Coplanar.

Performance level of students:

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
51	29	1	3	2	14	0.70

General Remarks :

80% of the candidates in the sample have either not attempted the question or have secured zero mark. Only about 26% of those who attempted this question obtained full marks. The concept of co-planarity is not clear to most of the candidates.

Errors Committed with examples :

Most of the candidates have directly inferred from the given relation

$$5\vec{a} - 3\vec{b} + 4\vec{c} - 6\vec{d} = 0$$

that points are linearly dependant and hence coplanar.

Probable Causes :

(i) Concepts of coplanarity and linear dependance of vectors is not clear to the candidates

(ii) Section formula, as applicable to vectors, is not clear to many of them.

Suggested Remedial Measures :

(i) Sufficient practice should be given in the use of section formula in vectors

(ii) The following concepts should be made clear and emphasised upon.

(a) Intersecting lines are always coplanar

(b) The points with position vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ are coplanar if there exist scalars α, β, r and δ such that $\alpha\bar{a}+\beta\bar{b}+v\bar{c}+\delta\bar{d}=0$ and $\alpha+\beta+v+\delta=0$

Q. No. 5. Verify Rolle's Theorem for the function $f(x)=x^2-6x+5$ in the interval [1, 5]

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Knowledge	Rolle's Theorem

Expected Answer :

(i) $f(x)=x^2-6x+5$ is continuous in [1, 5] being a polynomial in x.

(ii) $f'(x)=2x-6$, which exists $\forall x \in]1,5[$

(iii) $f(1)=f(5)=0$

$\therefore \exists$ a point $c \in]1,t[$, such that $f'(c) = 0$

i.e. $2c-6=0 \Rightarrow c=3$

$\because c=3 \in]1,5[$, Rolle's Theorem is verified.

Performance level of students:

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
5	7	4	12	15	57	1.59

General Remarks :

95% of the candidates in the sample have attempted this question and 60% of them secured full marks. Others committed conceptual as well as computational errors.

Errors Committed with examples :

(i) Conditions of Rolle`s theorem are neither stated nor verified

(ii) Concept of open and closed intervals has not been used correctly.

(iii) Some of the candidates have not verified whether 'c' belongs to open interval or not.

(iv) Even from simple linear equation of the type $2x - 6 = 0$, the value of x has not been calculated correctly.

Probable Causes :

(i) The candidates assume that the conditions of Rolle`s theorem are satisfied without actually verifying the same.

(ii) The candidates do not care to verify the domain of c.

(iii) Computational errors of very elementary nature are committed because of sheer negligence.

Suggested Remedial Measures :

(i) The students should be asked to verify the three conditions of Rolle`s Theorem.

(ii) Concept of closed and open intervals should be made more clear to them.

(iii) Sufficient practice needs to be given students to help them avoid computational erros.

Q. No. 6. Evaluate

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	limit of a function

Expected Answer :

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$$

Substituting $\frac{\pi}{2} - x = y$ so that when $x \to \frac{\pi}{2} \Rightarrow y \to 0$

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x = \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \cot y$$

$$= \lim_{y \to 0} \frac{y}{\tan y} = 1$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
20	41	3	10	1	25	0.79

General Remarks :

61% of the candidates in the sample have either not attempted or secured zero mark in this question. Only 31% of those who attempted could get full marks. The Mean score of the sample on this question is for below expectations.

Errors Committed with examples :

(i) The conditions under which the standard results of limit holds, are not clear to the candidates.

e.g. $\lim_{x \to 0} \frac{\tan x}{x} = 1$ but $\lim_{x \to \pi/2} \frac{\tan x}{x} \neq 1$

(ii) $\left(\frac{\pi}{2} - x\right)$ tan x has been written as $\frac{\tan \pi_2 - \tan x}{1 + \tan \pi_2 \tan x}$

some others have written this as $\pi/2 \tan x - \tan x^2$

(iii) $\lim_{x \to 0} x \cot x$ has been taken as zero in place of 1.

Probable Causes :

(i) Concept of limit of a function, in general, does not seem to be clear to many candidates.

(ii) Some of the candidates do not remember trigonometric formulae.

Suggested Remedial Measures :

(i) The concept of limit of a function needs to be made more clear to the students by giving various examples.

(ii) Sufficient practice needs to be given to the students in the use of different trigonometric formulae so that they can apply these correctly.

Q. No. 7. Differentiate $\tan^{-1}\left[\frac{1-\cos x}{\sin x}\right]$ w.r. t. x

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Knowledge	Differentiation of Inverse trigonometric function

Expected Answer :

$$y = \tan^{-1}\left[\frac{1-\cos x}{\sin x}\right]$$

$$y = \tan^{-1}\left[\frac{2\sin^2 \frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
5	16	4	4	2	69	1.56

General Remarks :

95% of the candidates in the sample attempted this question and about 72% of them got full marks. About 28% of those who attempted made conceptual errors because of lack of knowledge of trigonometric formulae.

Errors Committed with examples :

(i) $\tan\left(\frac{1-\cos x}{\sin x}\right)$ is taken as $\tan^{-1}(\operatorname{cosec} x) - \tan^{-1}(\cot x)$

(ii) (1–cos x) has been taken as $2\sin^2 x$

(iii) $\frac{d}{dx}\left(\tan^{-1}\frac{1-\cos x}{\sin x}\right)$ is taken as $\frac{1}{1+\left(\frac{1-\cos x}{\sin x}\right)^2}$

(iv) Some of the candidates simplified the expression $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ to $\frac{x}{2}$ but have not differentiated it.

Probable Causes :

(i) The candidates lack the knowledge of inverse trigonometric functions and knowledge of trigonometric formulae.

(ii) The derivative of $f[g(x)]$ has not been understood and done by many of the candidates.

Suggested Remedial Measures :

(i) Sufficient practice should be given to candidates in the use of trigonometric formulae.

(ii) Concept of inverse trigonometric functions should be made clear to the candidates with the help of examples.

(iii) Candidates should be given sufficient practice to find the derivative of composit functions.

Q. No. 8. Evaluate

$$\int \frac{\operatorname{cosec}^2 x}{1-\cot^2 x} dx$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	02	Knowledge	Integration (Integration by substitution

Expected Answer :

Put cot x = t $\Rightarrow \operatorname{Cosec}^2 x \, dx = -dt$

$$\therefore \int \frac{\operatorname{cosec}^2 x}{1-\cot^2 x} dx = -\int \frac{dt}{1-t^2} = \int \frac{dt}{t^2-1}$$

$$= \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| + C = \frac{1}{2} \log \left| \frac{\cot x - 1}{\cot x + 1} \right| + C$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
13	27	02	10	09	39	1.18

General Remarks :

87% of the candidates in the sample attempted this question but only 45% of them did it correctly A good number of candidates committed conceptual and computational errors

Errors Committed with examples :

(i) The derivative of cot x is taken as $\text{cosec}^2 x$.

(ii) $\int \frac{1}{a^2 - x^2} dx$ is taken as $\log \left| \frac{a+x}{a-x} \right|$ or $\frac{1}{a} \log \left| \frac{a-x}{a+x} \right|$

Probable Causes :

(i) The standard result on integration are not learnt properly by many of the candidates.

(ii) The standard results on differentation are not known to a significant number.

Suggested Remedial Measures :

(i) Sufficient practice needs to be given to find the derivative of those trigonometric functions which involve negative sign in the derivative.

(ii) Sufficient practice should also be given in problems involving the use of standard results on integration and differentiation.

Q. No. 9. Evaluate

$$\int e^x \sec x \,(1 + \tan x)\, dx$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Integration (i) By parts or (ii) by using the formula $\int [f(x) + f'(x) e^x dx = e^x f(x) + c$

Expected Answer :

Let $I = \int e^x \operatorname{Sec} x\,(1 + \tan x)\,dx$

$= \int (\sec x + \sec x \tan x)\, e^x\, dx$

$= e^x \sec x + c$ [By use of formula]

or Integrating by parts we get

$I = \sec x . e^x - \int e^x . \sec x \tan x\, dx + \int e^x \sec x \tan x\, dx + c$

$= e^x \sec x + c$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
10	15	10	4	–	61	1.46

General Remarks :

90% of the candidates in the sample attempted this question. Out of those who attempted, 68% did it correctly. The rest committed either conceptual or computational errors.

Errors Committed with examples :

(i) Many candidates used the following incorrect step :

$\int \sec x\,(1 + \tan x)\, e^x\, dx = \sec x \int e^x\, dx - \int \left[\int e^x\, dx\right] \int \sec x\, dx + c$

(ii) Some of the candidates who tried to solve the question by other method, failed to identify it with the standard result i.e.

$\int \left[f(x) + f'(x)\right] e^x\, dx = e^x f(x) + c$

Probable Causes :

The candidates lack practice in solving questions based on "Integration by Parts" or making use of standard result.

Suggested Remedial Measures :

Sufficient practice should be given to the candidates to solve questions based on "Integration by Parts" or based on the standard result of $\int[f(x)+f'(x)]e^x\,dn$

Q. No. 10. Evaluate $\int_0^6 |x-2|\,dx$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	knowledge	Definite Integrals involving the use of property $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Expected Answer :

$$\text{Let } I = \int_0^6 |x-2|\,dx$$

$$= \int_0^2 (2-x)\,dx + \int_2^6 (x-2)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^6 \quad = 2 + 8 = 10$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
17	33	2	10	8	30	1.00

General Remarks :

83% of the candidates in the sample attempted this question. 36% of those who attempted,did it correctly and 10% did not complete steps of solutions. Others committed computational or conceptual errors.

Errors Committed with examples :

(i) Some of the candidates solved the question ignoring modulus sign as follows.

$$\int_0^6 |x-2|\,dx = \int_0^6 (x-2)\,dx \text{ r } \int_0^6 (2-x)\,dx$$

(ii) Some of the candidates applied the property

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ incorrectly

Probable Causes :

(i) The concept of modulus/absolute value is not clear to many candidates.

(ii) The candidates were also not able to identify the point 'c' inbetween the limits 'a' and 'b' and as such could not apply the property

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Suggested Remedial Measures :

Adequate practice needs to be given to stndents in questions involving modulus functions.

Q. No. 11. Evaluate $\int \frac{dx}{\sqrt{x^2-2x+4}}$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Indefinite integrals based on $\int \frac{dx}{\sqrt{x^2+a^2}}$

Expected Answer :

$$I = \int \frac{dx}{\sqrt{x^2-2x+4}}$$

$$= \int \frac{dx}{\sqrt{(x-1)^2 + (\sqrt{3})^2}}$$

$$= \log \left| (x-1) + \sqrt{x^2 - 2x + 4} \right| + c$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
13	11	1	17	4	54	1.52

General Remarks :

87% candidates in the sample attempted this question. Out of those who attempled, 62% did it correctly and 7% did not give compete steps of solution. Others committed computational or conceptual errors.

Errors committed with examples :

(i) Some candidates have not been able to express the given expression as sum of two squares as $(x-1)^2 + (\sqrt{3})^2$

(ii) Other candidates applied the formula for

$\int \frac{dx}{x^2 + a^2}$ in place of $\int \frac{dx}{\sqrt{x^2 + a^2}}$

Probable Causes :

The candidates lack practice in reducing quadratic expressions to the standard forms whose integrals are known to them

Suggested remedial Measures :

(i) The difference between $\int \frac{dx}{\sqrt{x^2 + a^2}}$ and $\int \frac{dx}{x^2 + a^2}$ should be explained to the students with the help of examples

(ii) Sufficient practice should be given to the students for changing / writing a quadratic expression in the form of sum of two squares as used in the solution.

(iii) While using the formula of log $\left|x+\sqrt{x^2+a^2}\right|$ modulus sign must be put as log of negative quantity is not defined .

Q. No. 12. Evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x}dx$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Knowledge	Definite Integrals (Use of the property $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

Expected Answer :

Let $I=\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x}dx$ ....(i)

$=\int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)}dx$ (Using $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$)

$=\int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x+\sin^3 x}dx$ ....(ii)

adding (i) & (ii) to get.

$2I=\int_0^{\pi/2} dx = x\Big]_0^{\pi/2}=\pi/2$

$\Rightarrow I=\frac{\pi}{4}$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
22	3	3	1	5	66	1.82

General Remarks :

78% of the candidates in the sample attempted this question. 84% of those who attempted did it correctly. Others committed conceptual or computational errors.

Errors committed with examples :

(i) Some of the candidates tried to attempt this question directly by substitution without using the property of definite integrals which was not possible.

(ii) $\sin\left(\frac{\pi}{2}-0\right)$ has been taken as sin θ and $\cos\left(\frac{\pi}{2}-\theta\right)$ has been taken as cos θ

(iii) Incorrect application of the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Probable Causes :

(i) Candidates lack elementary knowledge of trigonometry such as $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$, etc.

(ii) Candidates lack the use of knowledge of application of the property

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Suggested remedial Measures :

(i) Correct use of trigonometrical results should be emphasised upon and sufficient practice should be given to apply the same.

(ii) Sufficient practice of questions based on the properties of definite integrals should be given to the students.

Q. No. 13. Two unbiased dice are thrown. Find the probability that the sum of the numbers obtained on the two dice is neither a multiple of 2 nor a multiple of 3.

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Probability (Involving mutually exclusive events)

Expected Answer :

Total number of possible outcomes = $6 \times 6 = 36$

Set of possible outcomes {(1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (4, 3) (5, 6), (6, 5), (3, 4)}

n(s) = 12

$\therefore$ Required probability $= \frac{12}{36} = \frac{1}{3}$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1½	2	
25	18	17	20	2	18	0.90

General Remarks :

75% candidates in the sample attempted this question, of which 24% did it correctly getting full marks. As many as 58% candidates committed conceptual errors.

Errors committed with examples :

(i) The candidates lack the concept of "mutually exclusive events" and, as a result have written incorrect number of favourable cases.

(ii) Some of the candidates wrote all the 36 possible outcomes which was not needed. Although it was not an error, it resulted in wastage of time.

Probable Causes :

(i) The candidates are not able to :

(a) identify the total number of outcomes.

(b) number of favourable outcomes.

Suggested remedial Measures :

(i) The students should be given practice to differentiate between favourable cases and the total sample.

(ii) The students should be asked to solve problems involving idea of mutually exclusive events.

Q. No. 14. Find the regression co-efficient of x on y for the following data:

$$\sum x = 32,\ \sum y = 24,\ \sum xy = 218,\ \sum x^2 = 216,\ \sum y^2 = 246,\ n = 8$$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	knowledge	Calculation of regression Coefficients

Expected Answer :

$$b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$$

$$= \frac{8 \times 218 - 32 \times 24}{8 \times 246 - (24)^2}$$

$$= \frac{976}{1392}$$

$$= 0.70$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
5	28	3	14	12	38	1.15

General Remarks :

95% of the candidates from the sample attempted this question but only 40% of them, could do it correctly. About 40% candidates committed computational errors while above 12% committed conceptual errors.

Errors committed with examples :

(i) Some candidates have used the formula for b_{x_y} as $b_{x_y} = \frac{\frac{1}{n}\sum xy - \sum x \sum y}{\frac{1}{n}\sum y^2 - (\sum y)^2}$, which gave wrong result.

(ii) Some other students confused regression co-efficint with correlation co-efficient and calculated the same. Some others calculated covarience in place of regression co-efficient.

(iii) Square roots have not been evaluated.

Probable Causes :

(i) Candidates lack knowledge of use of logarithmic tables for calculation.

(ii) Difference between co-efficient of regression and co-efficient of correlation is not clear to many candidates.

Suggested remedial Measures :

(i) Difference between regression co-efficient and correlation co-efficient needs to be made clear to the students. Individual significance of the two may be pointed out separately.

(ii) Difference between regression co-efficient of x on y and y on x should be made more clear to the students.

(iii) Sufficient practice should be given in finding b_{xy} and b_{yx} to eliminate computational errors.

(iv) Practice should be given to do calculations with the help of logarithm tables.

Q. No. 15. Solve the differential equation $\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$

Type of Question	Marks	Objective	Concept/Sub Concept
VSA	2	Understanding	Solution of Differential equations (variable Separable type.)

Expected Answer :

$$\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\cos x \sin y}{\cos y}$$

$$\Rightarrow \frac{\cos y \; dy}{\operatorname{Sin} y} = -\cos x \, dx$$

Integrating, we get

$$\log|\sin y| = -\sin x + c$$

$$\text{or } \log|\sin y| + \sin x = c$$

Performance level of students :

Number of students not attempting	Number of students getting marks					Mean score
	0	½	1	1 ½	2	
5	6	4	24	12	49	1.49

General Remarks :

95% of the candidates in the sample attempted this question. But only 52% did it correctly getting full marks. 15% candidates could not complete the question while others committed computational or conceptual errors.

Errors committed with examples :

(i) Many candidates could not separate the variables; some of them wrote as

(a) $\frac{dy}{dx} + \frac{\cos x \ \sin y}{\cos y} = 0$

or $\frac{\cos y}{\sin y} + \cos x = 0$

$\Rightarrow \log \ \sin y + c = -\sin x + C$

or $\log \ \sin y + \sin x = 0$

(ii) Integration symbol has not been used at some places.

(iii) Constant of integration has not been written

(iv) Some candidates wrote the given equation as $dy \cos y + dx \,.\cos x \sin y = 0$

Probable Causes :

(i) The candidates appear to have got mixed up as some of them took the equation as variable-separable type and others took it as homogeneous diff. equation.

(ii) Inadequate knowledge of trigonometry.

Suggested remedial Measures :

(i) Students should be given sufficient practice to identify differential equations of various types.

(ii) The students should be asked to remember standard results of integrals and use them in different differential equations.

(iii) Importance of constant of integration should be emphasized upon.

Q. No. 16. Jsing the properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Use of properties in evaluating determinants

Expected Answer :

$$R_1 \to R_1 - R_3 \Rightarrow (a+b+c) \begin{vmatrix} 1 & 0 & -1 \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

$$R_2 \to R_2 - R_3 \Rightarrow (a+b+c)^2 \begin{vmatrix} 1 & o & -1 \\ o & 1 & -1 \\ c & a & c+a+2b \end{vmatrix}$$

$$C_3 \to C_1 + C_2 + C_3 \Rightarrow 2(a+b+c)^3 \begin{vmatrix} 1 & o & o \\ o & 1 & o \\ o & o & 1 \end{vmatrix}$$

$$= 2\ (a+b+c)^3$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
3	5	0	4	1	4	0	1	1	81	3.54

General Remarks :

97% of the candidates from the selected sample attempted the question out of which about 8% committed conceptual errors. 83% candidates did the question correctly getting full marks. About 5% candidates committed computational errors.

Errors committed with examples :

(i) Operations performed are not clearly indicated.

(ii) There are instances where the candidates indicated column operation but performed row operation.

(iii) Instead of solving the determinant by using properties, expansion has been used at the earlier or at the later stage, which is not desired.

Probable Causes :

(i) The candidates carry out computational work carelessely.

(ii) Many of the candidates fail to differentiate between row/column operations on determinants.

Suggested remedial Measures :

(i) All operations on rows/columns should be clearly indicated.

(ii) Instead of using equality sign, 'arrows' should be used.

(iii) It should be explained to the students that if it is required to use the properties of determinants, then the expansion should not be used at any stage till the determinant is of the order 2.

(iv) Sufficient practice should be given to reduce/transform the given higher order determinant to lower order determinant using properties.

Q. No. 17. A plane passes throgh a fixed point (1,–2,3,) and cuts the axes in A,B and C. Show that locus of the centre of the sphere passing through the points O,A,B and C is given by $\frac{1}{x}-\frac{2}{y}+\frac{3}{z}=2$.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Equation of plane and Sphere and the Concept of Locus

Expected Answer :

Equation of plane through A (a,o,o), B (o,b,o), C (o,o,c)

is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ (Intercept form)

(1, -2, 3) lies on the plane $\Rightarrow \frac{1}{a}-\frac{2}{b}+\frac{3}{c}=1$.............(i)

Equation of sphere through O,A,B,C is $x^2+y^2+z^2-ax-by-cz=0$

Centre of Sphere is $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

If (x_1, y_1, z_1) is the Centre of sphere then $a=2x_1,\ b=2y_1,\ c=2z_1$

Substituting in (i) we get the eqn of locus as $\frac{1}{x}-\frac{2}{y}+\frac{3}{2}=z.$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
55	25	3	5	2	5	–	–	–	5	0.87

General Remarks :

80% of the candidates in the sample have either not attempted this question or secured zero mark. Only 5% candidates could solve the question completely getting full marks. Mean score of 0.87 of the sample (out of 4) shows that the candidates are not at all clear about the concepts related to locus, equations of plane and sphere.

Errors committed with examples :

(i) The point (1, –2, 3) lies on the sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$, which is meaningless

(ii) Equation of the plane through (1, –2, 3) is

a (x – 1) + b (y + 2) + c (z – 3) = 0

and (a, o , o), (o, b, o), (o, o, c) lie on it

Probable Causes :

The candidates do not seem to be clear about the concepts on locus, equations of plane and sphere which resulted in above type of errors.

Suggested remedial measures :

Sufficient practice should be given to the students in

(i) finding equations of planes in different situations

(ii) finding equations of spheres for different data

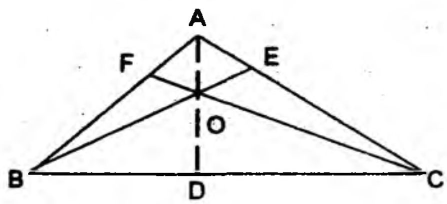
(iii) problems dealing with locus of points under given conditions.

Q. No. 18. Using vectors, prove that the altitudes of a triangle are concurrent.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Application	Vectors and perpendicularity

Expected Answer :

Let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of the points A,B,C, respectively. Draw $BE \perp AC$ & $CF \perp AB$ to meet at O. AO is joined & produced to meet BC in D taking O as origin,



$BE \perp AC \Rightarrow \bar{b}.(\bar{c}-\bar{a})=0$

$CF \perp AB \Rightarrow \bar{c}.(\bar{b}-\bar{a})=0$

$\therefore\ \bar{b}.\bar{c}-\bar{b}.\bar{a}=\bar{b},\bar{c}-\bar{c}.\bar{a}$

$\Rightarrow \bar{a}.(\bar{c}-\bar{b})=0$

$\therefore \vec{AO} \perp \vec{BC}$ or $\vec{AD} \perp \vec{BC}$

$\Rightarrow$ Altitudes are concurrent.

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
36	25	9	7	1	4	0	1	1	16	1.43

General Remarks :

61% candidates from the sample have either not attempted this question or secured zero mark. Only 25% of those who attempted could get full marks. Majority of students could not even start the solution.

Errors committed with examples :

(i) Most of the candidates committed mistake in the steps related to concept of perpendicularity.

(ii) Difference between altitude and median is not clear to many of the candidates.

(iii) Many candidates have drawn the figure showing three altitudes already meeting at a point.

Probable Causes :

(i) Concept of perpendicularity is not clear to many candidates

(ii) Difference between altitude, median, and angle bisectors is not clear to same of the candidates.

(iii) Candidates lack practice in applying the knowledge of vectors in proving geometrical results.

Suggested remedial Measures :

(i) Sufficient practice needs to be given to the students in using the knowledge of vectors for proving various geometrical results.

(ii) Difference between angle bisectors, altitudes, medians in a triangle should be made very clear to the students taking different types of triangles.

Q. No. 19. Find from first principle, the derivative of $\sqrt{\sec x}$ w. r. t. x.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	knowledge	Differentiation from first principle

Expected Answer :

$$y = \sqrt{\sec x}$$

$$y + \Delta y = \sqrt{\sec (x + \Delta x)}$$

$$\Delta y = \sqrt{\sec (x + \Delta x)} - \sqrt{\sec x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\sqrt{\sec (x + \Delta x)} - \sqrt{\sec x}}{\Delta x} \times \frac{\sqrt{\sec (x + \Delta x)} + \sqrt{\sec x}}{\sqrt{\sec (x + \Delta x)} + \sqrt{\sec x}}$$

$$= \frac{\sec (x + \Delta x) - \sec x}{\Delta x\left[\sqrt{\sec (x + \Delta x)} + \sqrt{\sec x}\right]} = \frac{[\cos (x + \Delta x) - \cos x]}{\cos (x + \Delta x) \cos x \, \Delta x\left[\sqrt{\sec (x + \Delta)} + \sqrt{\sec x}\right]}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 \sin \left(x + \frac{\Delta x}{2}\right)}{\cos (x + \Delta x) \cos x} \left(\frac{\sin \frac{\Delta x}{2}}{\frac{\Delta x}{2}}\right) \cdot \frac{1}{\sqrt{\sec (x + \Delta x)} + \sqrt{\sec x}}$$

$$\frac{dy}{dx} = \frac{\sec x \ \tan x}{2\sqrt{\sec x}}$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
4	6	1	13	3	12	3	2	3	53	2.90

General Remarks :

Though 96% of the candidates in the sample attempted this question, only about 55% could attempt it correctly. The other 45% candidates committed conceptual mistakes.

Errors committed with examples :

(i) Some candidates differentiated the function directly without using first principle.

(ii) $f'(x)$ is written as $\frac{f(x+h)-f(x)}{h}$

(iii) sec (x + h) is written as sec x + sec h

(iv) Some candidates have not done rationalization correctly.

(v) Others committed mistakes in use of trigonometric results.

Probable Causes :

(i) The process of finding derivative of a function ab-initio is not clear to the candidates.

(ii) Candidates seem to lack adequate knowledge of trigonometric results.

Suggested remedial Measures :

(i) Definiton of derivative should be made clear to the students and sufficient practice should be given to them to find the derivative of various function ab-initio.

(ii) The students should be made to learn different trigonometric results and their applications.

Q. No. 20. For the function $f(x)=2x^3-8x^2+10x+5$ find the intervals.

(a) In which f(x) is increasing

(b) In which f(x) is decreasing.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	4	Understanding	Increasing and decreasing functions

Expected Answer :

$$f(x) = 2x^3 - 8x^2 + 10x + 5$$

$$f'(x) = 6x^2 - 16x + 10$$

$$= 2(3x - 5)(x - 1)$$

$$f'(x) = 0 \Rightarrow x = 1,\ x = \frac{5}{3}$$

Intervals	(3x −5)	(x − 1)	$f'(x) = 2(3x-5)(x-1)$	Nature of function
$x < 1$	−ve	−ve	+ve	Increasing
$1 < x < \frac{5}{3}$	−ve	+ve	−ve	Decreasing
$x > \frac{5}{3}$	+ve	+ve	+ve	Increasing

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
7	3	9	7	5	9	1	6	6	47	2.66

General Remarks :

93% of the candidates in the sample attempted this question. Of those who attempted, only 2.2% secured zero mark and about 50% completed the question getting full marks.

Errors committed with examples :

(i) The candidates did not factorise the quadratic polynomial obtained after finding the derivative of the given function.

(ii) Some of the candidates appear to have the misconception that when f'(x) < 0, f(x) is increasing and it is decreasing when f'(x) > 0

(iii) For checking whether the function is increasing or decreasing, many candidates put the stationary values in the given function in place of making some suitable intervals and checking the behavior of f(x) in those intervals.

Probable Causes :

(i) Lack of practice of factorisation of quadratic polynomials and quadratic inequations.

(ii) Concept of increasing and decreasing functions is not clear to many.

Suggested remedial Measures :

(i) Sufficient practice needs to be given to the students in factorising quadratic polynomials and solving quadratic inequations.

(ii) Concept of increasing and decreasing functions needs to be made clear to the students through various examples and through geometrical interpretation.

Q. No. 21. Draw a rough sketch and find the area of the region bounded by the two parabolas $y^2 = 8x$ and $x^2 = 8y$, by using method of integration.

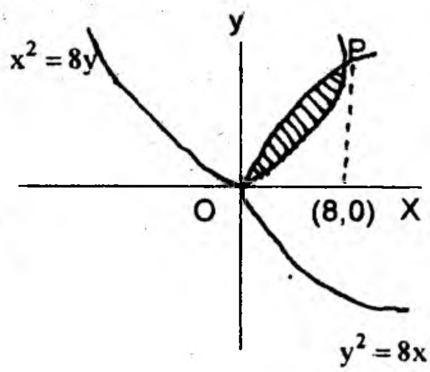
Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	(to find area under the curves) Using integration

Expected Answer :

$\therefore$ x -coordinate of the point of intersection = 8

$$\text{Area} = \int_0^8 \left(2\sqrt{2}\,\sqrt{x} - \frac{1}{8}x^2\right) dx$$

$$= 2\sqrt{2}.\frac{x^{3/2}}{3/2} - \frac{1}{8}\cdot\frac{x^3}{3}\Bigg|_0^8 = \frac{64}{3} \text{ Sq units}$$



Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
29	11	03	09	01	13	04	08	02	20	2.21

General Remarks :

71% candidates out of the given sample attempted this question. Out of these who attemped, 28% did it correctly, 17% did not give complete steps of solution and others committed computational and conceptual errors.

Errors committed, with examples :

(i) Graphs of the curves $y^2 = 8x$ and $x^2 = 8y$ were not properly drawn

(ii) Some of the candidates did not shade the required region bounded by the two parabolas.

(iii) Others calculated the area as $\int_0^8 (\frac{1}{8}x^2 - 2\sqrt{2}\sqrt{x})\,dx$ which resulted in negative answer.

(iv) Point of intersection of the curves was not calculated correctly.

(v) Units of area were not mentioned.

Probable Causes :

(i) The candidates do not possess the skill of plotting rough sketch of the given equation.

(ii) Some of them lack practice in solving the equations of two curves to find their point of intersection.

Suggested remedial Measures :

(i) The students need to be given sufficient practice to draw rough sketches of graph, of the type $y^2 = ax$ and $x^2 = by$, and shade the required region.

(ii) Sufficient practice should be given to find the point of intersection of two curves.

(iii) Writing proper units of quantities should be emphasised upon.

Q. No. 22. Find the value of $\int_0^3 (2x^3+3)\,dx$ as limit of sums

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Integration (integration as limit of sums)

Expected Answer :

$I = \int_0^3 (2x^2+3)\,dx$ Here : a = 0, b = 3,

$$\therefore h = \frac{3-0}{n} = \frac{3}{n}$$

$$I = \lim_{n\to\infty} \frac{3}{n}\left[f(o)+f(h)+f(2h)+......+f(n-1\,h)\right]$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[3+(2h^2+3)+(8h^2+3)+......+[2(n-1)^2 h^2+3]\right]$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[3n+\frac{18}{n^2}\cdot\frac{n(n-1)(2n-1)}{6}\right]$$

= 27

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
11	5	01	03	05	18	04	09	–	44	2.92

General Remarks :

89% of the candidates in the sample attempted this question out of which 49% did it correctly. 12% candidates did not give complete steps of solution, 24% and 15% committed computational and conceptual errors respectively.

Errors Committed with examples :

(i) Some of the candidates did not remember correct formula for limit of the sums

(ii) Others used the result $h = \frac{b-a}{n}$ incorrectly and calculated it as $= -\frac{3}{n}$

(iii) Still others did not remember the result $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ Some of them incorrectly for finding $\sum (n-1)^2$ as $\frac{n(n-1)(2n+1)}{6}$

Probable Causes :

(i) Lack of practice in writing the formula for limit of the sum.

(ii) Lack of knowledge of the formulae $\sum n, \sum n^2$ and $\sum (n-1)^2$

Suggested Remedial Measures :

(i) The students should be given sufficient practice in using the value of h as $h = \frac{b-a}{n}$

(ii) They should be asked to learn the formula for evaluating the limit of sums correctly.

(iii) The candidates should be made to practise rigorously the integrals as limit of the sums of the type $\int_a^b e^x \, dx$, $\int_p^q (ax+b) \, dx$ $\int_r^s (ax^2+bx+c) \, dx$

Q. No. 23. Evaluate : $\int \frac{dx}{(2-x)(x^2+3)}$

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Integration (using partial fraction)

Expected Answer :

$$\text{let } \frac{1}{(2-x)(x^2+3)} = \frac{A}{(2-x)} + \frac{Bx+C}{x^2+3}$$

$$\therefore A = \frac{1}{7},\ B = \frac{1}{7},\ C = \frac{2}{7}$$

$$\therefore \int \frac{dx}{(2-x)(x^2+3)} = \frac{1}{7}\int \frac{dx}{2-x} + \frac{1}{7}\int \frac{(x+2)}{x^2+3}\,dx$$

$$= -\frac{1}{7}\log|2-x| + \frac{1}{14}\log|x^2+3| + \frac{2}{7\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
23	17	03	08	05	09	04	03	04	24	2.13

General Remarks :

77% of the candidates out of the sample attempted this question, out of which 31% did it correctly, 9% did not give complete steps of solution, 32% and 28% committed computational and conceptual errors respectively.

Errors Committed with examples :

(i) Some of the candidates have written the expression

$$\frac{1}{(2-x)(x^2+3)} = \frac{A}{2-x} + \frac{B}{x^2+3}$$

(ii) Majority of the candidates committed computational errors in finding the values of A,B and C.

(ii) Some of the candidates have written $\int \frac{dx}{1-x} = \log(1-x)$

Probable Causes :

(i) Candidates lack practice in resolving the given expression into partial fractions.

(ii) Some of them lack computational skill due to lack of practice.

(iii) Many of them lack in basic concepts of integration.

Suggested Remedial Measures :

(i) Sufficient practice needs to be given to the students in splitting the expressions into partial fractions.

(ii) Sufficient practice should be given to the students in evaluating simple basic integrals.

Q. No. 24. Two balls are drawn at random from a bag containing 3 white, 4 green and 4 black balls, one by one without replacement. Find the probability that both the balls are of different colours.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Probability (independent event)

Expected Answer :

P (both balls of different colours)

$= 1 - P$ (both balls of same colour)

$$= 1 - \left[\frac{3C_2}{14C_2} + \frac{3C_2}{14C_2} + \frac{4C_2}{14C_2} + \frac{4C_2}{14C_2}\right]$$

$$= \frac{73}{91}$$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1½	2	2½	3	3½	4	
35	24	03	04	03	09	03	03	01	15	1.66

General Remarks :

65% candidates in the sample attempted this question. Out of them, 23% did it correctly, 17% did not give complete steps of solution, 20% committed computational errors & others committed conceptual errors.

Errors Committed with examples :

(i) The concept of finding the probability of independent events is confused with dependent events.

(ii) Some of the candidates evaluated probability of getting both balls of same colour but did not subtract it from 1 to get the required probability.

(iii) Some others gave the answer of probability as greater than 1.

Probable Causes :

(i) Candidates fail to differentiate between independent and dependent events, while finding the probability.

(ii) Same of the candidates lack in knowledge that sum of all the probabilities pertaining to an event is 1.

(iii) Many of them lack in knowledge on meaning and use of n_{c_r}

Suggested Remedial Measures :

(i) The concept of n_{c_r} and its application must be made clear to the students by giving different examples.

(ii) Sufficient practice needs to be given to the students in problems based on independent and dependent events so that they can differentiate between them.

Q. No. 25. Two cards are drawn successively with replacement from a deck of 52 cards. Find the probability distribution of the number of aces drawn.

Type of Question	Marks	Objective	Concept/Sub Concept
SA	04	Understanding	Probability (Binomial Probability distribution)

Expected Answer :

Here $n=2$, $p=\frac{1}{13}$, $q=1-p=\frac{12}{13}$

$$P(o)=2C_0\left(\frac{1}{13}\right)^0\left(\frac{12}{13}\right)^2=\frac{144}{169}$$

$$P(1)=2C_1\left(\frac{1}{13}\right)^1\left(\frac{12}{13}\right)^1=\frac{24}{169}$$

∴ **Required Probability distribution** : $P(2)=2C_2\left(\frac{1}{13}\right)^2=\frac{1}{169}$

x	0	1	2
P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Performance level of students :

Number of students not attempting	Number of students getting marks									Mean score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	
39	25	03	06	–	03	–	–	01	23	1.80

General Remarks :

61% of the candidates in the sample attempted this question. Out of which about 38% candidates did it correctly getting full marks. 11% candidates did not give complete solution and others either committed computational errors or conceptual errors.

Errors Committed with examples :

(i) Many candidates failed to identify the question properly, due to inadequate knowledge of probability distribution theory.
theory.

(ii) Some candidates found p as $\frac{1}{4}$ and q as $\frac{3}{4}$

(iii) Others could not evaluate the expression, $n_{c_r} p^r q^{n-r}$

for different values of p,q and r

(iv) Only few candidates wrote the probability distribution in tabular form

Probable Causes :

(i) Many candidates do not understand the basic concepts of probability and probability distribution.

(ii) Many of them lack the skill of calculating binomial co-efficients $n_{c_r} p^r q^{n-r}$ for various values of r, p and q.

Suggested Remedial Measures :

(i) The students need to be explained the concept of probability clearly through various examples

(ii) The conditions under which the binomial distribution is applicable should be discussed in detail.

Q. No. 26. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}

Using A^{-1}, solve the following system of linear equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	Solving System of Linear Equations using inverse of a matrix.

Expected Answer :

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = -1$$

$$\text{Adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

using $x = A^{-1} B$

$x = 1, \ y = 2, \ z = 3$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
1	3	1	1	2	4	1	11	2	14	-	12	1	47	4.65

General Remarks :

This is the only question where the response is almost 100% However, only 47% of the selected sample could do the question correctly. As many as 19% candidates committed conceptual errors while 40% made computational errors.

Errors Committed with examples :

(i) A majority of students used symbol of determinants instead of matrix AdjA is written as $\begin{vmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{vmatrix}$ and $x = A^{-1} B$ as $(-1) \begin{vmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{vmatrix} \begin{vmatrix} 11 \\ -5 \\ 3 \end{vmatrix}$

(ii) Some candidates have committed computational errors while finding the value of the determinant and adjoint. In case of an adjoint, the +ve and - ve signs are not taken care of.

(iii) Some candidates left the question incomplete after finding A^{-1}.

(iv) Cramer's rule has been used to solve the linear equation by a significant number of candidates.

Probable Causes :

(i) Candidates do not read the question properly. This is indicated by the fact as some of the candidates have solved the question using cramer's rule.

(ii) Many candidates have committed computational errors.

(iii) Carelessness about signs while taking adjoints.

Suggested Remedial Measures :

(i) Use of proper notations for determinant and matrix should be emphasised.

(ii) Sufficient practice should be given to find the adjoint of a given matrix.

(iii) Students should be asked to read the question carefully and solve the question by the method specified in the question paper.

(iv) Sufficient practice be given to develop computational skills.

Q. No. 27. Define the line of shortest distance between two skew lines. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by

$\vec{r} = 6\hat{i} + 3\hat{k} + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

and $\vec{r} = 9\hat{i} + \hat{j} - 10\hat{k} + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Type of Question	Marks	Objective	Concept/Sub Concept
LA	06	Understanding	Shortest distance betwen skew lines, to find the point of intersection and equation of line.

Expected Answer :

Def. The line perpendicular to both the skew lines is the line of shortest distance.

Points $P(2\lambda+6, -\lambda, 4\lambda+3)$, $Q(4\mu-9, \mu+1, 6\mu-10)$ **on lines** L_1 **and** L_2

DR's of $\vec{PQ} < 4\mu - 2\lambda - 15,\ \mu + \lambda + 1,\ 6\mu - 4\lambda - 13 >$

$\vec{PQ} \perp \vec{L_1} \Rightarrow 21\lambda - 31\mu + 83 = 0 \cdots\cdots (i)$

$\vec{PQ} \perp \vec{L_2} \Rightarrow 31\lambda - 53\mu + 137 = 0 \cdots\cdots (ii)$

Solving (i) and (ii), are get $\lambda = -1,\ \mu = 2$

$\Rightarrow P(4, 1, -1),\ Q(-1, 3, 2)$

$\left|\vec{PQ}\right| = \sqrt{38}$

Vector equation of line is $\vec{r} = 4\hat{i} + \hat{j} - \hat{k} + \lambda\ (5\hat{i} - 2\hat{j} - 3\hat{k})$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1½	2	2½	3	3½	4	4½	5	5½	6	
08	2	1	2	2	8	15	34	2	14	3	4	1	4	3.11

General Remarks :

92% candidates in the sample attempted this question. of which only about 4% could complete the question getting full marks, 96% of those who attempted, did not give full steps of solution. Majority of the candidates atempted only that part of the question in which shortest distance is to be calculated using a particular formula.

Errors Committed with examples :

(i) Many students defined the line of shortest distance between two skew lines wrongly.

(ii) Some of the candidates made computational error in calculating

$\bar{b}_1 \times \bar{b}_2 \cdot (\bar{a}_2 - \bar{a}_1)$ and $\left|\bar{b}_1 \times \bar{b}_2\right|$

(iii) Some of the candidates could not convert equation from cartesion form to vector form.

Probable Causes :

(i) The concept of line of shortest distance is not clear to many candidates

(ii) The candidates do not remember the formula for calculation of shortest distance between two skew lines.

(iii) The candidates cannot convert equations given in cartesian form to vector form and vice-versa.

Suggested Remedial Measures :

(i) The students should be explained the concept of line of shortest distance between two skew lines with the help of examples and teaching aids.

(ii) The students should be helped to remember the formula for the line of shortest distance between two skew lines by asking them to put in more practice.

(iii) They should be given sufficient practice in converting equations in vector form to cartesian form and vice-versa

Q. No. 28. A wet porous substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet hung in the wind loses half its moisture during the first hour when will it have lost 90% moisture, weather conditions remaining the same?

Type oi Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	Differential equation

Expected Answer :

Let m_0 be the moisture content initially and m be the moisture content after t hours

$$\frac{dm}{dt} \alpha\, m \Rightarrow \frac{dm}{dt} = Km.$$

$$\Rightarrow \int \frac{dm}{m} = \int K\, dt \Rightarrow 10gm = Kt + C$$

at $t = 0$, $m = m_0 \Rightarrow \log m_0 = c$

$\Rightarrow \log m = Kt + \log m_0$

at $t = 1$ hr $m = m_o/2 \Rightarrow \log \frac{mo}{2} = K + \log m_o \Rightarrow K = -\log 2$

Putting $m = \frac{1}{10} m_0$

$$\Rightarrow \log \frac{m_0}{10} = -t \log 2 + \log m_0 \Rightarrow -t \log 2 = \log \frac{m_0}{m_0 0/10} = -\log 10$$

$$\Rightarrow t = \frac{\log 10}{\log 2}$$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1½	2	2½	3	3½	4	4½	5	5½	6	(%)
63	11	–	3	–	5	–	5	2	2	1	0	2	6	95

General Remarks :

The performance of the sample in this question is extremely poor as only 6% candidates could attempt it correctly. 31% students failed to understand the question while 63% did not attempt it.

Errors Committed with examples :

(i) Translation is not done properly. Candidates failed to put the verbal information into mathematical equation.

(ii) Loosing 90% moisture is taken as 90% of the moisture i.e. $M = \frac{90}{100} M_o$.

(iii) Constant of proportionality i.e. k is considered as moisture content.

(iv) Variables are not properly defined

(v) Constant of integration is not taken.

Probable Causes :

(i) Word problems are not understood properly

(ii) Inadequate proficiency in translation of verbal information into mathematical equations.

(iii) Lack of knowledge of solving differential equations especially finding the value of constant of integration under given physical conditions.

Suggested Remedial Measures :

(i) Sufficient practice should be given to translate word problems into mathematical equations.

(ii A thorough understanding of solving differential equations should be provided to the students.

(iii) The importance of constant of integration in differential equations is to be emphasised in relation to given physical conditions.

Q. No. 29. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Application	Application of derivatives in problems of Maxima & Minima

Expected Answer :

Let the dimensions of closed cuboid be x, x and y

$\Rightarrow V = \text{Volume} = x^2y$

A= Surfare area of the cuboid $= 2(cb + bh + he)$

$= 2(x.x + xy + xy)$

$= 2x^2 + 4xy$

$$\Rightarrow A = 2x^2 + \frac{4v}{x}$$

$$\frac{dA}{dx} = 4x - \frac{4v}{x^2}$$

For area to be Maximum or minimum

$$\frac{dA}{dx} = 0 \Rightarrow 4x - \frac{4v}{x^2} = 0 \Rightarrow V = x^3 \Rightarrow \text{Cuboid is a cube.}$$

$$\frac{d^2A}{dx^2} = 4 + \frac{8v}{x^3} > 0 \Rightarrow \text{Surface area is minimum}$$

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
39	20	1	5	–	5	2	4	–	2	–	4	–	18	2.77

General Remarks :

The question based on application of differential calculus is poorly attempted. As many as 59% candidates in the sample have either not attempted the question or have obtained zero.

Errors Committed with examples :

(i) Translation of word problem into mathematical formulations is not done properly.

(ii) Surface area is taken as $6(lb + bh + hl)$ or lbh.

(iii) Volume of cuboid is taken as $\frac{1}{3}\pi r^2 h$.

(iv) Instead of finding $\frac{dA}{dx}$, $\frac{dv}{dA}$ has been calculated

(v) Second derivative has not been determined.

Probable Causes :

(i) Candidates are not able to transform verbal statements into mathematical formulations.

(ii) Candidates do not remember the standard results of mensuration.

(iii) Concept of maxima/minima is not clear to many of the candidates.

Suggested Remedial Measures :

(i) A good practice to translate verbal statements into mathematical equations is desired

(ii) Standard results of mensuration should be learnt and understood by the students.

(iii) Concept of maxima/minima should be made more clear.

Q. No. 30. Find Karl pearson's co-efficient of correlation between x and y for the following data:

x :	10	8	11	14	12	10	11	13
y :	12	10	13	15	13	14	13	15

Also give the interpretation of the coefficient of correlations thus obtained.

Type of Question	Marks	Objective	Concept/Sub Concept
LA	6	Skill	Karl Pearson's coefficient of correlation -calculation of p(x,y)

Expected Answer :

If assumed means in x series is 11 and that of y series is 13, then

$$\sum dx = 1, \sum dy = 1, \sum dx\, dy = 19, \sum dx^2 = 25, \sum dy^2 = 19$$

$$p(x_1 y) = \frac{n\sum dx\, dy - \sum dx \sum dy}{\sqrt{n\sum dx^2 - (\sum dx)^2}\sqrt{n\sum dy^2 - (\sum dy(^2)}}$$

Correct Substitution of values and getting P(x, y) = 0.87

Interpretation :

High degree of positive correlation i.e. x increases (or decreases with increase (or decrease) *in* y and vice-versa.

Performance level of students :

Number of students not attempting	Number of students getting marks													Mean Score
	0	½	1	1 ½	2	2 ½	3	3 ½	4	4 ½	5	5 ½	6	
2	2	–	–	1	1	5	17	8	23	19	17	4	1	3.94

General Remarks :

98% of the candidates from the sample attempted this question, out of which 10% committed conceptual errors. Out of those who attempted only 1.02% candidates could do the question correctly to the end. About 56% candidates committed computational errors which is not expected of candidates from senior secondary stage.

Common errors with examples :

(i) More than 60% of the candidates have left the question incomplete. They have not given either the interpretation of the result obtained or have left the question as such without calculating the final answer.

(ii) Wrong formula used.

Some candidates have used the formula

$$P(x_1 y) = \frac{\frac{1}{n}\sum dx\,dy - \sum dx \sum dy}{\sqrt{\frac{1}{n}\sum dy^2 - (\sum dy)^2}\sqrt{\frac{1}{n}\sum dy^2 - (\sum dx)^2}}$$

which has given wrong results.

(iii) In step deviation method; wrong computation 8-11= 3 instead of -3.

(iv) In setting of table, calculaltion errors have been committed.

(v) P(x, y.) has been calculated to be more than 1.

Probable Causes :

(i) Since many candidates do not understand the meaning and significance of P(x, y) =0.87 etc, they are not able to give the interpretation of result.

(ii) Many candidates do not remember the formula.

(iii) Correct formula for deviation is not known to many.

Remedial Measures :

(i) Interpretation of the result should be made clear while teaching the topic.

(ii) Practice may be given to make calculations with the help of logarithm tables.

(iii) The students should be asked to learn respective formulae again and again so that they may not forget the same.,

(iv) It should be made clear that the value of p'(x,y) lies between -1 and +1.

TOPICWISE STATISTICAL SUMMARY OF STUDENTS' PERFORMANCE.

(Outside Delhi 65/1)

Matrices and Determinants.

Sl. No. of Question	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
1	93	28	13	13	6	33	1.02	51.0
2	98	7	3	10	1	77	1.74	87.0
16	97	5	4	05	1	82	3.54	88.5
26	99	3	4	16	16	60	4.65	77.50

In the unit on Matrices and Determinants, overall performance of candidates has been satisfactory. Each question has been attempted by more than 90% candidates, the least attempt (93%) is in No 1 and the maximum attempt being in Q. No 26. The mean score on this unit ranges from 51% on Q. No. 1 to 88.5 % in Q No 16.60% of those who attempted Q.No. 1 scored marks below 50% whereas more than 80% of those who attempted the question got more than 75% marks in Nos. 2 and 16. The poorest performance has been in Q. No. 1 with an average score of 51% and the best performance has been in Q. No. 2 with an average score of 87%.

Vectors and 3-Dimensional Geometry

Sl. No. of Question	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
03	83	11	7	3	17	45	1.47	73.5
04	49	29	1	3	2	14	0.70	35.0
17	45	25	8	7	–	5	0.87	21.75
18	64	25	16	5	1	17	1.43	35.75
21	92	2	5	76	19	9	3.11	51.83

In the unit on vectors and 3-Dimensional Geometry, the best performance has been in Q. No. 3, with a mean score of 73.5% and worst performance has been in Q. No. 17 with a mean score of 21.75%.

Q. Nos. 17 and 4 have been found to be most difficult in this unit, with an average (mean) percentage scores of 21.75% and 35% respectively. On all other questions, the mean scores range from 35% to 52%

Q. Nos. 4 and 17 have been found to be more difficult by the students as less than 50% candidates tried to attempt this question.

Differential calculus

Question Number	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
5	95	7	4	12	15	57	1.60	80.0
6	80	41	3	10	1	25	0.79	39.5
7	95	16	4	4	2	69	1.55	78.0
19	96	6	14	15	5	56	2.90	72.5
20	93	3	16	14	7	53	2.66	66.5
29	61	20	6	11	2	22	2.77	46.17

In the unit on Differential calculus, the best performance has been in Q. No. 5 (80%) followed by mean percentage score of 78% in Q. No. 7. For all other questions, the mean percentage score ranges from 39.5% in no 6 to 72.5% in Q. No. 19.

In Q. Nos 5.19 and 20, more than 60% of the candidates (who attempted the question) secured more than 75% marks whereas In Nos 5 and 29, more than 60% of the candidates (who attempted the question) obtained 50% or less marks.

Integral Calculus :

Question Number	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
8	87	27	02	10	09	39	1.2	60.0
9	90	15	10	04	–	61	1.46	73.00
10	83	33	2	10	08	30	1.00	50.00
11	87	11	1	17	04	54	1.52	76.0
12	78	03	03	01	05	66	1.82	91.0
21	71	11	12	14	12	22	2.21	55.25
22	89	05	04	23	13	44	2.92	73.0
23	77	17	11	14	07	28	2.13	53.25

In the topic on integral calculus, almost all the questions have been attempted by more than 70% candidates, the largest (90%) being in Q. No. 9 and the least (71%) being in Q. NO. 21. The least mean percentage score has been in Q. NO. 10, followed by 53.25% in Q. No. 23 and 55.25% in Q. No. 21.

More than 70% of the candidates, who attempted this question, got 75% or more marks in Q. No.'s 12 and 9

This percentage has been in the range of 30% and 50% in case of other questions, with the exception of Q. NO. 11 for which about 65% candidates secured 75% or more marks.

Differential Equations

Question Number	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
15	95	6	4	24	12	49	1.50	75.0
28	37	11	3	10	5	8	2.57	42.83

In the unit on Differential equations, Q. No. 15 has been attempted by majority of the candidates with a mean score of 75%. More than 66% of the candidates attempting this question got more than 50% marks.

The other question (No. 20) has been attempted by only 37% candidates with an average score of 42.83% More than 66% candidates, who attempted this question, secured 50% or less marks. This is due to the reason that it was a word problem testing the objective of "application" which is expected to be covered by "above average candidates."

Correlation and Regression

Question Number	No. of candidates attempting the question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
14	95	28	3	14	12	38	1.16	58.0
30	98	2	1	23	50	22	3.94	65.67

In the unit on correlation and regression, both the question Nos. 14 and 30 have been found to be popular with the students as 95% and 98% candidates (respectively) have attempted these questions, with mean percentage scores of 58 and 65.67 respectively.

In Q. No. 14, quite a good percentage of students (about 30%) secured zero mark whereas this figure for Q. No. 30 is only about 2%.

In Q. No. 14, 53% candidates got more than 50% marks whereas the corresponding figure for Que. No. 30 is 74%.

Probability

Question Number	No. of candidates attempting the Question	Number of Candidates getting marks					Mean Score	Mean % Score
		0	1-25%	26-50%	51-75%	76-100%		
13	75	18	17	20	2	18	0.90	45.0
24	65	24	07	12	6	16	1.66	41.50
25	61	25	09	03	00	24	1.80	45.50

The performance of candidates in this unit on probability has been below 50% on all the questions. Q. No. 13 has been attempted by 75% candidates of which only 26% secured more than 75% marks. Q. Nos 24 and 25 have been attempted by 65% and 61% candidates respectively of which 25% and 40% obtained 75% or more marks for Q. Nos 24 and 25 respectively.

OVERALL STATISTICAL SUMMARY FOR CODE 65/1

Q. No.	No. of cand. attempting the Q	0	Number of Candidates getting marks ½	1	1½	2	2½	3	3½	4	4½	5	5½	6	Mean Score	Mean % Score
1	93	2	13	13	6	33									1.02	51.0
2	88	7	3	10	1	77									1.74	87.0
3	83	11	7	3	17	45									1.47	73.5
4	49	29	1	3	2	14									0.70	35.0
5	95	7	4	12	15	57									1.60	80.0
6	88	41	3	10	1	28									0.79	39.5
[illegible]	[illegible]	16	[illegible]	[illegible]	[illegible]	[illegible]									1.55	78.0
[illegible]	87	27	[illegible]	[illegible]	[illegible]	38									1.2	60.0
[illegible]	[illegible]	15	10	[illegible]	[illegible]	61									1.46	73.0
10	63	35	2	10	8	30									1.00	50.0
11	87	11	1	17	4	54									1.52	76.0
12	78	3	3	1	5	66									1.82	91.0
13	78	16	17	20	2	18									0.90	45.0
14	85	28	3	14	12	38									1.16	58.0
15	95	6	4	24	12	49									1.50	75.0
16	97	5	0	4	1	4	0	1	1	81					3.54	88.5
17	45	25	3	5	2	5	–	–	–	5					0.87	21.75
18	64	26	9	7	1	4	–	1	1	16					1.43	35.75
19	96	6	1	13	3	12	3	2	3	53					2.90	72.5
20	93	3	9	7	5	9	1	6	6	47					2.66	66.5
21	71	11	3	9	1	13	4	8	2	20					2.21	55.25
22	89	5	1	3	5	18	4	9	–	44					2.92	73.0
23	77	17	3	8	5	9	4	3	4	24					2.13	53.25
24	65	24	3	4	3	9	3	3	1	15					1.66	41.50
25	61	25	3	6	–	3	–	–	1	23					1.80	45.00
26	99	3	1	1	2	4	1	11	2	14	–	12	1	47	4.65	77.50
27	92	2	1	2	2	8	15	34	2	14	3	4	1	4	3.11	51.83
28	37	11	–	3	–	5	–	5	2	2	1	–	2	6	2.57	42.83
29	61	20	1	5	–	5	2	4	–	2	–	4	–	18	2.77	46.17
30	98	2	–	–	1	1	5	17	8	23	19	17	4	1	3.94	65.67

TOPICWISE SPECIFIC COMMON MISTAKES AND SUGGESTED REMEDIAL MEASURES

Though specific errors and mistakes committed by majority of students in every question have been given in the preceeding section of this document, it would be more fruitful to highlight topicwise common mistakes committed by them at one place. This would help the teachers to lay more stress on related teaching/learning points while dealing with different content areas. Suggested remedial teaching measures included in this section,it is hoped, would further enhance effectiveness of teaching.

Matriçes and Determinants

The study reveals that many of the students do not use correct notation for a determinant or a matrix. A significant number of these students could not carry out proper row and column transformation and the operation of multiplication on the same. The fact that many students use expansion of determinant at very early stage instead of using different properties to calculate its value is an indication that they do not clearly understand the use of these properties. A sizable percentage of students could not use appropriate method for determination of inverse of a Matrix and some even used column transformation instead of row transformation for the same purpose.

Providing sufficient practice to students in the use of correct notation and proper use of symbols needs to be given due attention. Sufficient practice should be provided to an average and below average student to minimise computational errors. Different methods to find inverse of a matrix should be explained clearly with special emphasis on avoiding column transformations for the same. Row and column transformations should be clearly indicated using arrows and not equalities. It should be emphasised upon that in order to find the value of a determinant, it is always better to transform higher order determinant to lower order determinant rather than straight way expanding it.

Vectors and Three - Dimensional Geometry

The analysis indicates that a significant number of students do not mark an arrow on the vectors correctly which generally leads to incorrect results. The concepts of dot and cross product of vectors and its application to unfamiliar situations appears to be a weak link in the understanding of the topic. Some of the students committed the mistake of using the condition of collinearity of three points in proving coplanarity of three vectors. Majority of the students did not understand the definition of line of shortest distance between two skew lines and either used incorrect formula or made computational mistakes in finding equation of the same. A large number of students could not attempt the question on locus correctly which indicates feeble and fragile learning in this area.

In order to promote better understanding of the concepts, the students need to be given sufficient practice on addition and subtraction of vectors by taking examples of vectors in different directions. The meaning and conditions of perpendicular vectors, parallel vectors and coplanar vectors should be explained by taking adequate number of examples. Since majority of the students did not perform well in questions related to locus, the concept needs utmost attention and detailed explanation in the classroom. Adequate practice also needs to be provded to the students in questions related to finding the equation of lines, plane and sphere with different possible given conditions.

Integral Calculus

It is observed that while some of the students did not write constant of integration 'c' in the final answer of a definite integral, others did not use the correct notation $\int f(x)\,dx$ and invariably wrote $\int f(x)$. Many students did not carry out the operation of partial fractions correctly. Incorrect application of the properties of definite integrals, incorrect use of result of trigonometric results $\sin\left(\frac{\pi}{2}-\theta\right)$ and $\cos\left(\frac{\pi}{2}-\theta\right)$, incorrect application of the results $\sum n^2$ in the evaluation of $\sum(n-1)^2$ are some of the common errors committed by many students in the sample. A sizable number of students could not draw the desired curves correctly and failed to highlight the area to be calculated.

Lot of emphasis needs to be given to the learning and understanding of trigonometric relations learnt in earlier classes. Thorough understanding of technique of partial fractions on the part of students demands greater focus and attention. Besides the knowledge of properties of definite intergrals and their use in questions of integration alongwith their proof should be clearly explained to the students. Sufficient practice in the use of results $\sum n$ and $\sum n^2$ to calculate the result of the type $\sum(n-1)^2$ will help the students in better understanding and solving the questions on integration by limit of sums. Drawing graphs of the curves and shading the desired regions for calculation of required area should be practised carefully. It should be brought home to the students that they must write constant of integration in solving definite integrals. They must also write dx etc. along with the function to be integrated.

Differential Calculus

The analysis reveals that the concept of limit has not been understood clearly by large percentage of students in the sample. A majority of them failed to attempt the question on differentiation of composite functions and lacked in sufficient knowledge of inverse function and standard trigonometric functions. Many of them, it is observed,did not clearly understand inequalities and hence failed to attempt the question on increasing and decreasing function. Some of them do not appear to be well versed in handling trigonometric derivative without taking the limit. Translation of word problems into mathematical equations and conditions for maximum and minimum value for a given function also need to be stressed upon. Computational errors at many steps is an indicator of element of negligance on the part of learners.

Review and revision of basic concepts and relations of algebra and trigonometry should be undertaken before teaching this topic in the class. The inequalities like $ab \geq 0$ and $ab \leq 0$ should be discussed thoroughly. A good practice in translation of word problems in mathematical equations and differentiating compsite functions is desired in order to help the students in improvement of their academic attainment. Greater attention to computation can help them further in minimising their shortcomings.

Differential Equations

The analysis indicates that a significant number of students in the sample failed to translate verbal statements into mathematical equations. Many of them could not discriminate between homogeneous equation of second degree and equation with separable variables. Whereas some of them did not use the results of standard integrals correctly, others did not take into consideration the constant of integration while solving differential equations. Many of them did not use the given conditions correctly for calculating constant of integration.

Sufficient opportunities may be provided to the students to acquire the ability of translating verbal statements into mathematical equations. Importance and need of using constant of integration, wherever need to be, should be emphasised upon. Adequate practice in solving different types of differential equations and finding the value of constant of integration under given conditions should be given utmost attention.

Probability

The fact that the mean percentage score of the sample in all the questions from this unit is around 40-50% is an indicator that the students did not comprehend related concepts. The study reveals that many of the students lacked in understanding the concept of mutually exclusive events and an independent event. The value of $^{u}_{r}$ used in the question was not properly calculated by some of the students. Others did not even know that the probability of an event cannot be more than one. It is observed that presentation of probability distribution in tabular form is missing in the responses of most of the students.

Greater attention, and emphasis needs to be given to the clarification of basic concepts of independent events and mutually exclusive events. More stress nees to be given on application of conditions under which binomial distribution formula is applicable. Students should be made to solve sufficient number of problems to develop the computational skill in evaluating $n_{c_r}\ p^r\ q^r$ etc.

Correlation and Regression

The findings of the analysis reveal that majority of students in the sample used wrong formula for coefficient of regression. Many of them do not seem to be clear about the difference between regression coefficient of x on y and coefficient of regression of y on x. Whereas some of them confused this with coefficient of correlation, others calculated co-variance for the data. Many students used wrong formula for correlation coefficient as

$$\frac{\sum xy - \sum x \sum y}{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}$$

It is surprising to observe that while calculating deviations from a given number, some of the students even committed glaring mistakes such as 8-11=3 and 12-8=4 etc., which is an indicator of extreme carelessness. Some of the candidates have calculated the coefficient of correlation to be more than 1.

It is suggested that while teaching regression lines, the difference between the two regression lines (y or x and x or y) and their corresponding regression co-efficients should be made very clear to the students. Different formulae for finding the co-efficient of correlation and their suitability for different situations should be made clear by taking different examples. Sufficient practice needs to be given to the students to calculate b_{xy} and b_{yx} and co-efficient of correlation. This will help them build confidence and the number of errors committed by them will automatically reduce.

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